NEUTRINO MASS MODELS AND θ_{13}

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Model building in two pages

features of fermion mass spectrum

1) $N_g=3$ — all ν experiments but LSND expained by 3 ν_a so far [MiniBooNE]

2) hierarchies $m_\nu << m_f$ — most plausible explanation: L violation [$0\nu\beta\beta$]

more in detail

quarks

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1$$

$$|V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

leptons

$$\frac{m_e}{m_\tau} << \frac{m_\mu}{m_\tau} << 1 \qquad \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} << 1$$

perhaps less pronounced in neutrino sector

$$|U_{e3}| < 1$$

the only element of U_{PMNS} still allowed to be small

$$\xi_i \equiv \text{small parameters}$$

in modern model building we have two ways of understanding $|\xi_i| << 1$

ξ_i are small breaking terms of an approximate flavour symmetry

[Froggatt, Nielsen 1978]

when $\xi_i \to 0$ the theory becomes invariant under a flavour symmetry F

example $m_e \to 0$ in QED

$e \to e^{i\alpha} e \qquad e^c \to e^{-i\alpha} e^c$

I will mainly concentrate on this, in this talk

very appealing approach, unfortunately freedom is huge

symmetries global or local
continuous or discrete

breaking terms from SSB,
from radiative corrections,
ad-hoc explicit breaking

no compelling model from data at the moment

general questions

-- can we establish some general result, independent on details of model building?

-- how can data help in uncovering the right picture?

[more on this later on…]

ξ_i are small due to geometry

$\Lambda_F \equiv$ scale of flavour physics [unknown at present]

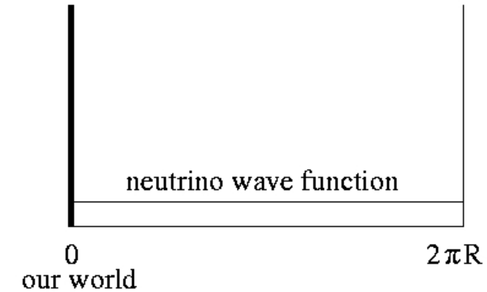
$E \approx \Lambda_F$ a four-dimensional description of particle interactions might break down

extra dimension could be tiny

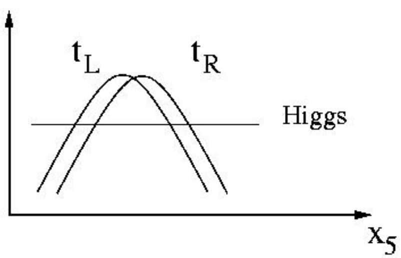
- Large Extra Dimensions

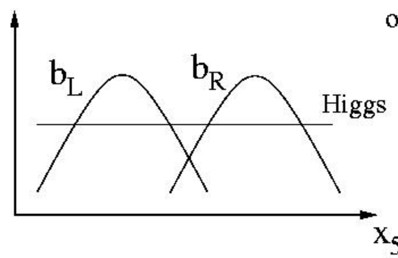
$$\frac{y_{\nu_e}}{y_e} \approx \frac{1}{\sqrt{2\pi R\Lambda}} << 1 \quad \text{if} \quad R >> \frac{1}{\Lambda}$$

flat zero mode for ν^c



- localized fermion zero modes





- Yukawa coupling in string theory

matter as twisted states in orbifold compactification of heterotic string

matter from intersecting D branes in type IIA strings

$$y_{ij} = e^{-A_{ij}}$$

Ibanez; Hamidi, Vafa; Dixon, Friedan, Martinec, Shenker; Casas, Munoz; Cremades, Ibanez, Marchesano; Abel, Owen

-- it could provide an explanation to $N_g = 3$

-- relation to flavour symmetries possible but not straightforward

-- connection with data unclear, worth to explore

ξ_i small due to the anthropic principle ?

too many models. Here: try to classify models by their predictions

Present and (near) future sensitivities

[Strumia, Vissani 0503246, Gonzales-Garcia 0410030, Maltoni, Schwetz, Tortola, Valle 0405172]

	current precision	future < 10 yr
Δm_{12}^2	$(8.0 \pm 0.3)\times 10^{-5}$ eV^2 [$\approx 4\%$]	few percent [KamLAND]
$\lvert\Delta m_{23}^2\rvert$	$(2.5 \pm 0.3)\times 10^{-3}$ eV^2 [$\approx 12\%$]	0.15×10^{-3} eV^2 LBL conventional beams 0.05×10^{-3} eV^2 [$\approx 2\%$] superbeams
ϑ_{12}	$\tan^2\vartheta_{12} = 0.45^{+0.05}_{-0.05}$ $\vartheta_{12} = (33.9\ ^{+1.4}_{-1.6})^0$	$\delta\tan^2\vartheta_{12} \approx 2\delta\sin^2\vartheta_{12}$ down by about a factor 2: challenging ν_e scattering rate of pp neutrinos to 1%
ϑ_{13}	$< 0.23\,(13^0)$ 90% C.L.	0.10 rad LBL, ChoozII 0.05 rad superbeams
ϑ_{23}	$\sin^2\vartheta_{23} = 0.52^{+0.07}_{-0.08}$ $\vartheta_{12} = 46^{0}{}^{+4^0}_{-5^0}$	$\delta\sin^2\vartheta_{23} \approx \delta\vartheta_{23}$ down by about a factor 2 superbeams
sign Δm_{23}^2	---	> 10 yr [Donini, Fernandez,
δ	---	> 10 yr Rigolin 0411402]

$$\vartheta_{13} \approx \delta\vartheta_{23} \approx \lambda^2 \approx 0.04 \div 0.05\,\text{rad}\,(2.1^0 \div 2.9^0)$$

significant level of precision for model building

most of existing models predict

$$\vartheta_{13} >> \lambda^2$$

$$\left|\frac{\pi}{4} - \vartheta_{23}\right| >> \lambda^2$$

``normal'' models

only in ``special'' models either of these conditions is violated

define:

``tiny'' ϑ_{13} $\Longleftrightarrow$ $\vartheta_{13} < \lambda^2$

``maximal'' ϑ_{23} $\Longleftrightarrow$ $\left|\frac{\pi}{4} - \vartheta_{23}\right| < \lambda^2$

normal models: some examples

-- degenerate spectrum

- anarchy [Hall, Murayama, Weiner 2000 De Gouvea, Murayama 0301050 Nir, Shadmi 2004]

$$m_\nu = m \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}$$

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} << 1$$

can be produced in part by the see-saw

ϑ_{13} tiny — accidental

ϑ_{23} maximal — fortuitous

- flavour democracy [Fritzsch, Xing]

$$\underset{f \neq \nu}{m_f} \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \ldots$$

$$m_\nu \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \ldots$$

$$\sin^2 2\vartheta_{23} = \frac{8}{9} \approx 0.89 \text{ (out by about } 2\,\sigma \text{ now)}$$

$$\vartheta_{13} \approx \sqrt{2m_e/3m_\mu} \approx (0.03 \div 0.1)$$

[estimates by allowing 3σ exp. and (factors ½ and 2) th. uncertainties]

inverted hierarchy

- the best we have, at present

$$m_\nu = m\begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix} + \ldots$$

corrections << |a|, |b|

- leading order determined by $L_e - L_\mu - L_\tau$ either with or without see-saw

- leading order

$$|m_1| = |m_2| = \sqrt{|a|^2 + |b|^2} \qquad m_3 = 0$$

$$\vartheta_{13} = 0$$

good!

$$\tan\vartheta_{23} = -\frac{b}{a}$$

large θ_{23} expected, maximal only by a fine-tuning

$$\vartheta_{12} = 45^o$$

it's out by 6σ

$$\vartheta_{12}^{\exp} = 33^o \pm 2^o$$

- turning SB terms on

$$1 - \tan^2\vartheta_{12} \approx O\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$$

$$0.36 \div 0.70\ (3\sigma) \quad >> \quad 0.015 \div 0.07\ (3\sigma)$$

off by a factor > 10

- substantial contribution to ϑ_{12} from charged leptons needed

$$U_{PMNS} = U_e^+ U_\nu$$

standard parametrization $U_e = U_{23}^e \cdot U_{13}^e \cdot U_{12}^e$

by expanding to 1st order in $|u| \equiv \sin\vartheta_{12}^e,\ |v| \equiv \sin\vartheta_{13}^e << 1$

$\vartheta_{23} \approx 45^o$

$$1 - \tan^2\vartheta_{12} = 2\sqrt{2}\,\mathrm{Re}(u+v)$$

$$|U_{e3}| = \frac{1}{\sqrt{2}}|u-v|$$

[Frampton, Petcov, Rodejohann 0401206
Altarelli, F, Masina 0402155
Romanino 0402508]

$$\tan^2\vartheta_{23} = 1 + O(u^2, v^2, uv)$$

$$\delta_{CP} = \arg(u-v)$$

if, by analogy with the quark sector:

$$|v| << |u| \approx \vartheta_C \approx 0.22$$

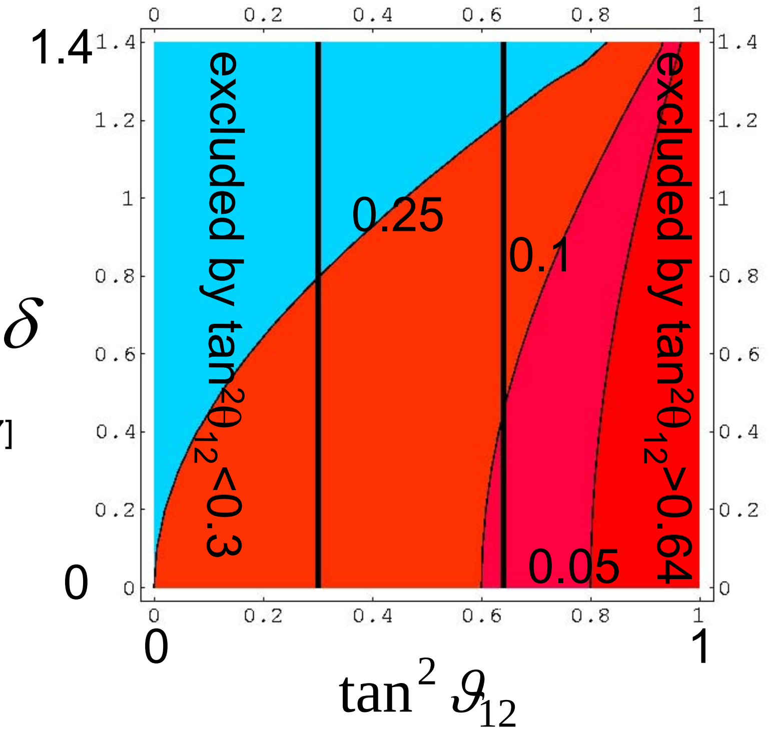
- $1 - \tan^2\vartheta_{12} \approx 2\sqrt{2}\,\vartheta_C \approx 0.6$

  [right amount]

  [Raidal 0404046
  Minakata, Smirnov 0405088
  Antush, King, Mohapatra 0504007]

- $$|U_{e3}| = \frac{(1-\tan^2\vartheta_{12})}{4\cos\delta_{CP}}$$

$\theta_{13} > 0.1$ expected



Normal Hierarchy

- Several viable mechanisms for ϑ_{23} large
  - ϑ_{23}^e and ϑ_{23}^ν small but $\vartheta_{23} \equiv \vartheta_{23}^\nu - \vartheta_{23}^e \approx O(1)$
  - see-saw dominance of light ν^c equally coupled to ν_μ and ν_τ [King]
  - lopsided structure of m_e or/and m_D^ν : $\bar{R}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & a & b \end{pmatrix} L$ [Albright, Barr Altarelli, F]

large θ_{23} expected, maximal only by a fine-tuning

$$U_{e3} \approx \sin\vartheta_{13}^\nu - \sin\vartheta_{23} \cdot \sin\vartheta_{12}^e$$

[1st order in $\sin\vartheta_{12}^e >> \sin\vartheta_{13}^e$]

ν-dominated

$$\approx \sin\vartheta_{12}^\nu \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}} \approx (0.03 \div 0.3)$$

e-dominated

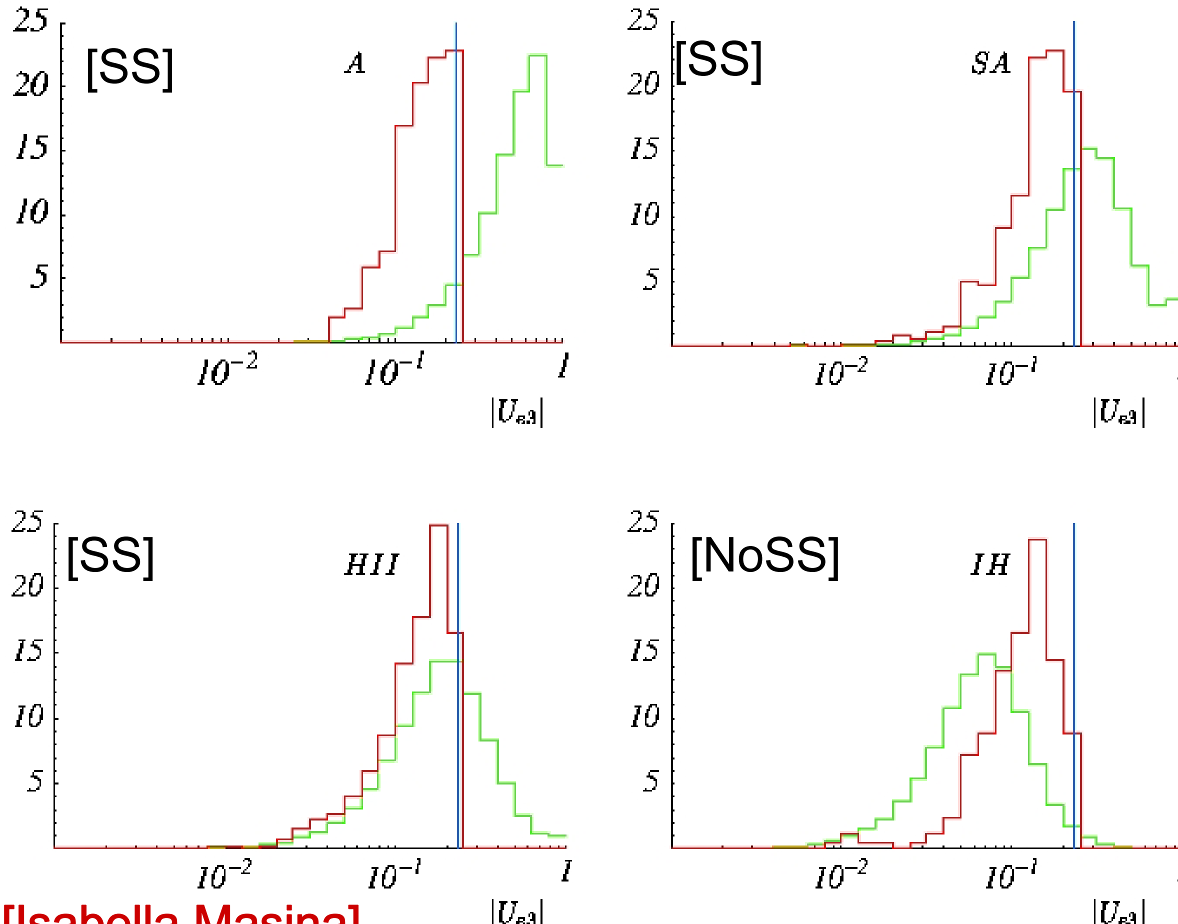
$$\approx -\sin\vartheta_{23} \sqrt{\frac{m_e}{m_\mu}} \approx (0.02 \div 0.1)$$

if we make a similar estimate in the quark sector

$V_{ub} \approx \lambda^3 \quad (\lambda \approx 0.22)$

θ_{13} not tiny, barring cancellations

U_{e3} in models with U(1) flavour symmetry



[Isabella Masina]

$$\begin{cases} 0.018 < r \equiv \left|\dfrac{\Delta m_{12}^2}{\Delta m_{23}^2}\right| < 0.053 \\ |U_{e3}| < 0.23 \\ 0.30 < \tan^2 \vartheta_{12} < 0.64 \\ 0.45 < \tan^2 \vartheta_{23} < 2.57 \end{cases}$$

ε optimised case by case to fit

$$m_\nu = m \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix}$$

$\varepsilon = 1$ anarchy=A

$\varepsilon < 1$ semianarchy=SA

$\varepsilon < 1$ normal hierarchy=H

$\det 23 \approx \mathcal{E}$

inverse hierarchy=IH

$\varepsilon < 1$

$$m_\nu = m \begin{pmatrix} \varepsilon^2 & 1 & 1 \\ 1 & \varepsilon^2 & \varepsilon^2 \\ 1 & \varepsilon^2 & \varepsilon^2 \end{pmatrix}$$

matrix elements up to unknown O(1) coeff.

Why $\vartheta_{13} < \lambda^2$ and $\left|\frac{\pi}{4} - \vartheta_{23}\right| < \lambda^2$ are difficult to achieve?

$\vartheta_{23} = \frac{\pi}{4}$ $\vartheta_{13} = 0$

of great help in our search for a unifying principle…

in some symmetry limit?

[RGE do not work. Casas, Espinosa, Navarro 0306243]

$$m_\gamma = 0 \quad \leftrightarrow \quad U(1)_{em} \text{ gauge symmetry}$$

$$m_e = 0 \quad \leftrightarrow \quad U(1) \text{ chiral symmetry}$$

$$\rho \equiv \frac{\mathrm{m_W}}{\mathrm{m_Z}\cos\vartheta_W} = 1 \quad \leftrightarrow \quad SU(2) \text{ custodial symmetry}$$

If the symmetry is **realistic**

$\vartheta_{23} = \frac{\pi}{4}$ $\vartheta_{13} = 0$ can never arise in the limit of exact symmetry

charged lepton mass matrix

$$m_l = m_l^0 + \ldots$$

symmetric limit (m_l^0)

symmetry breaking effects ($\ldots$): vanishing when flavour symmetry F is exact

similarly for neutrinos

realistic m_l^0 has rank ≤1

$$m_l^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

→ ϑ_{12}^e undetermined

$$U_{PMNS} = U_e^+ U_\nu$$

[omitting phases]

$$\tan \vartheta_{23}^0 = \tan \vartheta_{23}^\nu \cos \vartheta_{12}^e + \left(\frac{\tan \vartheta_{13}^\nu}{\cos \vartheta_{23}^\nu} \right) \sin \vartheta_{12}^e$$

$$\sin \vartheta_{13}^0 = \sin \vartheta_{13}^\nu \cos \vartheta_{12}^e - \cos \vartheta_{13}^\nu \sin \vartheta_{23}^\nu \sin \vartheta_{12}^e$$

undetermined

$\vartheta_{13} = 0$ $\quad \vartheta_{23} = \frac{\pi}{4}$

determined entirely by breaking effects (different, in general, for ν and e sectors)

if symmetry breaking is **spontaneous** ⇒

vacuum alignment problem

$$\langle \varphi_\nu \rangle, \langle \varphi_e \rangle, \dots$$

should have specific magnitudes and relative directions in flavour space.

requirements for ``special'' models based on a SB flavour symmetry

- alignment should be natural

  no ad-hoc relations
  desired VEVs from most general V
  in a finite region of parameter space

- alignment not spoiled by sub-leading terms

$$\vartheta_{13} = 0 + a_1 \frac{\langle\varphi\rangle}{\Lambda} + a_2 \frac{\langle\varphi\rangle^2}{\Lambda^2} + ...$$

$$\vartheta_{23} = \frac{\pi}{4} + b_1 \frac{\langle\varphi\rangle}{\Lambda} + b_2 \frac{\langle\varphi\rangle^2}{\Lambda^2} + ...$$

leading order

from higher-dimensional operators compatible with gauge and flavour symmetries

often $\frac{\langle\varphi\rangle}{\Lambda} \approx \lambda$
then $a_1 = b_1 = 0$ needed
for special models

- alignment compatible with mass hierarchies

$$\frac{m_e}{m_\tau}, \quad \frac{m_\mu}{m_\tau}$$ should vanish in the limit of exact symmetry

example [only lepton sector]

[Altarelli, F. 0504165]

controls charged lepton mass hierarchies

flavour symmetry

$$A_4 \times \ldots$$

group of even permutations of four objects

[Ma, Rajasekaran 2001; Babu, Ma, Valle 2003; Hirsch, Romao, Skandage, Valle, Villanova de Moral 2003; Ma 0409075]

	l	e^c	μ^c	τ^c	φ_e	φ_ν	ξ_ν	ϑ_e
A_4	3	1	1'	1"	3	3	1	0

$$\langle\varphi_e\rangle = (v,v,v)$$
$$\langle\varphi_\nu\rangle = (v',0,0)$$
$$\langle\xi_\nu\rangle = u$$

$$v \approx v' \approx u \approx \langle\vartheta\rangle$$

$$m_l = \begin{pmatrix} y_e & y_e & y_e \\ y_\mu & y_\mu\omega & y_\mu\omega^2 \\ y_\tau & y_\tau\omega^2 & y_\tau\omega \end{pmatrix} v_d \left(\frac{v}{\Lambda}\right) + \ldots$$

$$\omega \equiv e^{i\frac{2\pi}{3}}$$

$$m_\nu = \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix} \frac{v_u^2}{\Lambda} + \ldots$$

$$a \equiv y_a \frac{u}{\Lambda} \qquad d \equiv y_d \frac{v'}{\Lambda}$$

higher orders in $\frac{\mathrm{VEV}}{\Lambda}$

$$\vartheta_{13} \approx O\left(\frac{\mathrm{VEV}}{\Lambda}\right)^2 \qquad \vartheta_{23} \approx \frac{\pi}{4} + O\left(\frac{\mathrm{VEV}}{\Lambda}\right)^2 \qquad \tan\vartheta_{12} = \frac{1}{\sqrt{2}} + O\left(\frac{\mathrm{VEV}}{\Lambda}\right)^2$$

$\frac{\mathrm{VEV}}{\Lambda} \approx \lambda$ expected

[alignment is natural and it is not spoiled by higher dim operators]

ν spectrum is between normal and degenerate

$m_1 \approx -a \quad m_2 \approx a \quad m_3 \approx -3a$ [units $\frac{v_u^2}{\Lambda}$]

prediction:

$$|m_3|^2 = |m_{ee}|^2 + \frac{10}{9}\Delta m^2_{atm}\left(1 - \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$$

SUMMARY

- U_{e3} is special: the only element of U_{PMNS} still allowed to be small

- a joint experimental effort can greatly improved U_{e3} in a near future

- aimed for sensitivities $\vartheta_{13} \approx \delta\vartheta_{23} \approx \lambda^2 \approx 0.04 \div 0.05$
  might provide a significant
  progress in theory

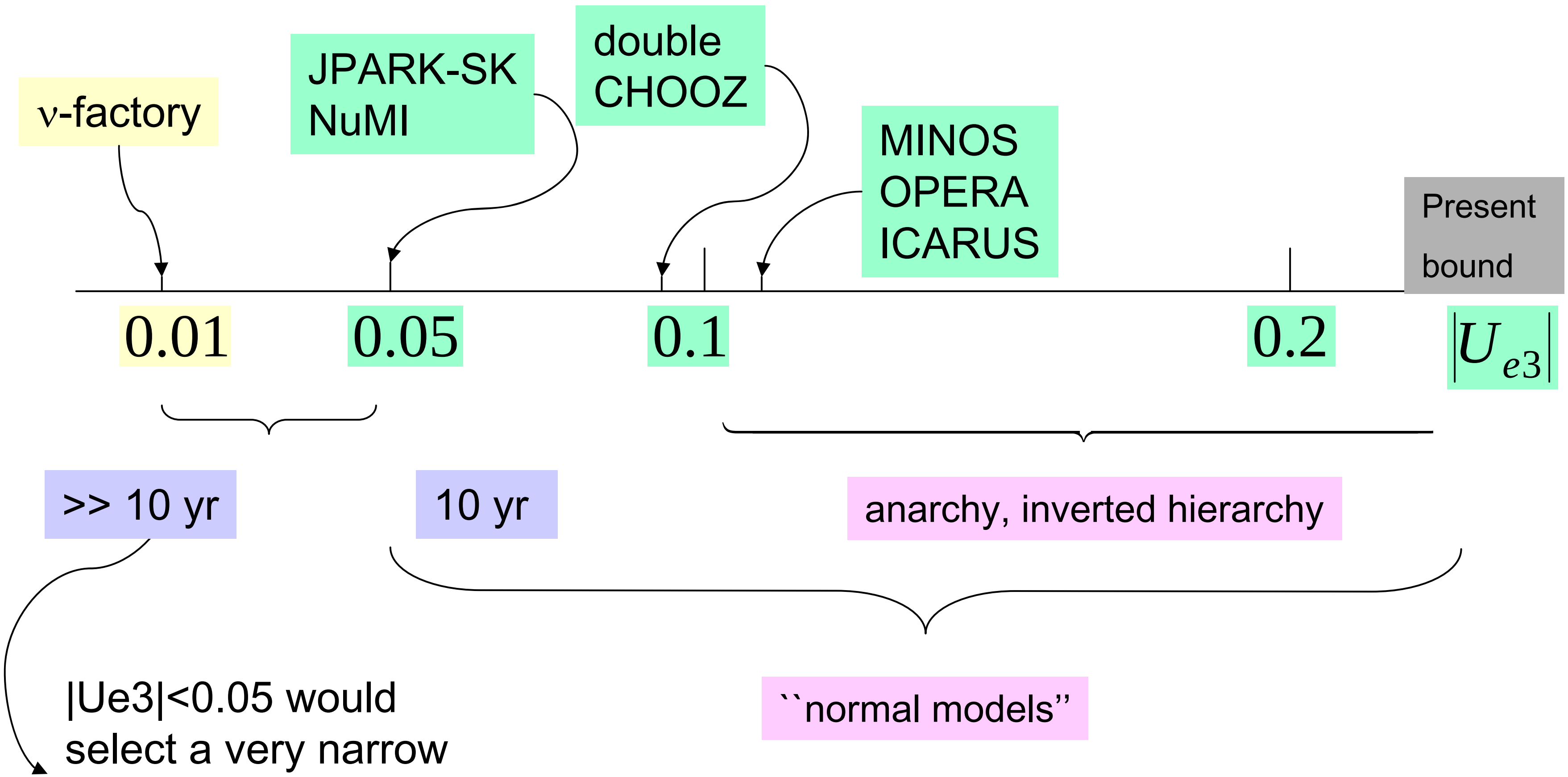
  most of existing models predict $\vartheta_{13} >> \lambda^2$ $\left|\frac{\pi}{4} - \vartheta_{23}\right| >> \lambda^2$

$\vartheta_{13} < \lambda^2$

$\left|\frac{\pi}{4} - \vartheta_{23}\right| < \lambda^2$

``special models''
e.g. SB flavour symmetry
-- natural vacuum alignment
-- special suppression of subleading contributions

- Most of plausible range for Ue3 explored in 10 yr from now



|Ue3|<0.05 would select a very narrow (not empty) subset of existing models

similar conclusion by:
Barbieri, Hambye, Romanino 0302118
Ibarra, Ross 0307051
Chen, Mahanthappa 0305088
Lebed, Martin 0312219
Joshipura @ NOON 2004