GDR Neutrino

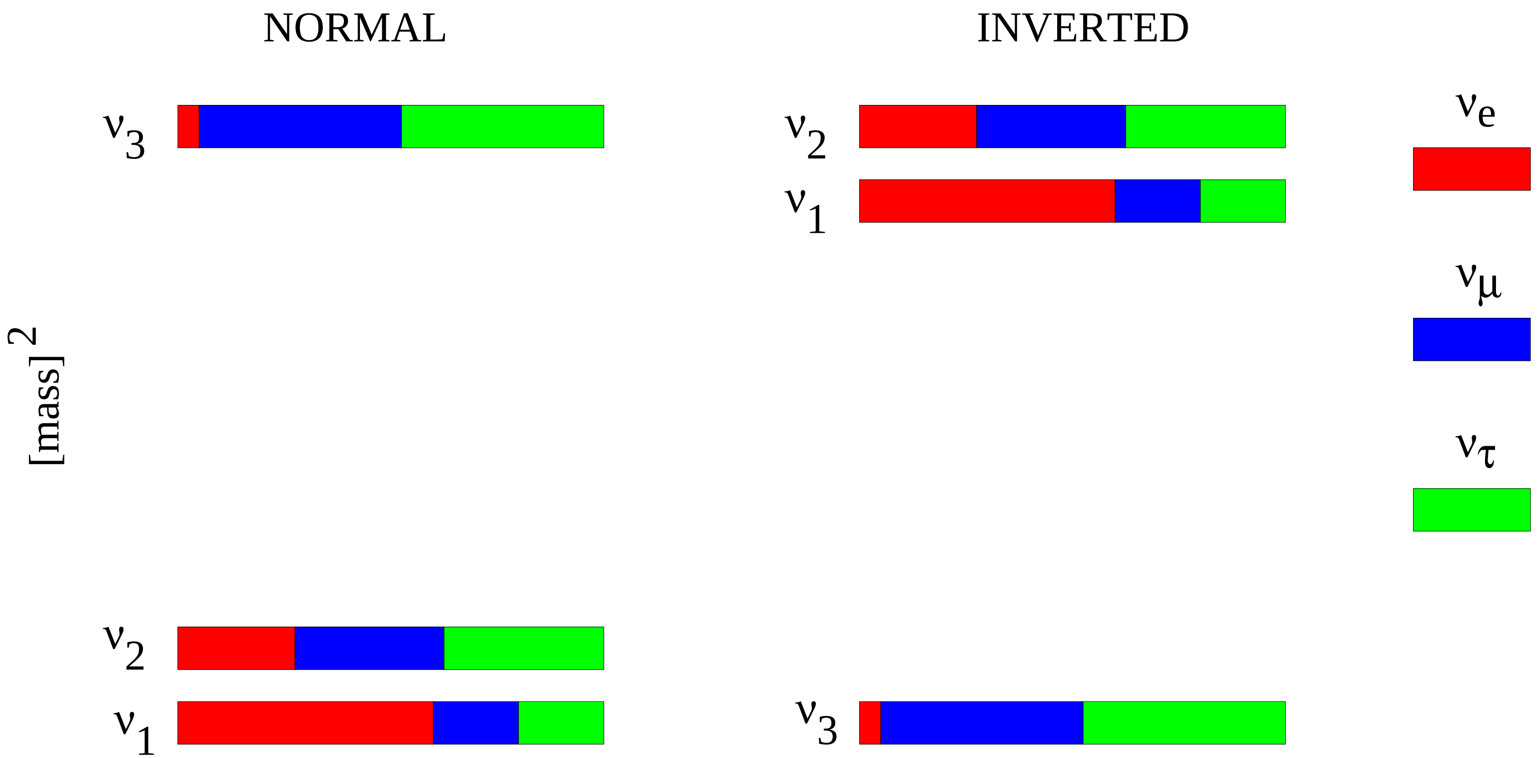
Determination of the neutrino mass hierarchy

Thomas Schwetz

CERN

The "hierarchy problem"

two possibilities for the neutrino mass spectrum



$\Delta m^2_{31} > 0$ $\Delta m^2_{31} < 0$

The "hierarchy problem"

The key to resolve the neutrino mass hierarchy is the matter effect!

Other possibilities do exist in principle...

- medium-baseline reactor experiments
- comparison of ν_e and ν_μ disappearance experiments
- neutrinoless double-beta decay

...but are extreemly difficult in practice.

- the same is true also for supernova neutrino observations

Outline of this talk

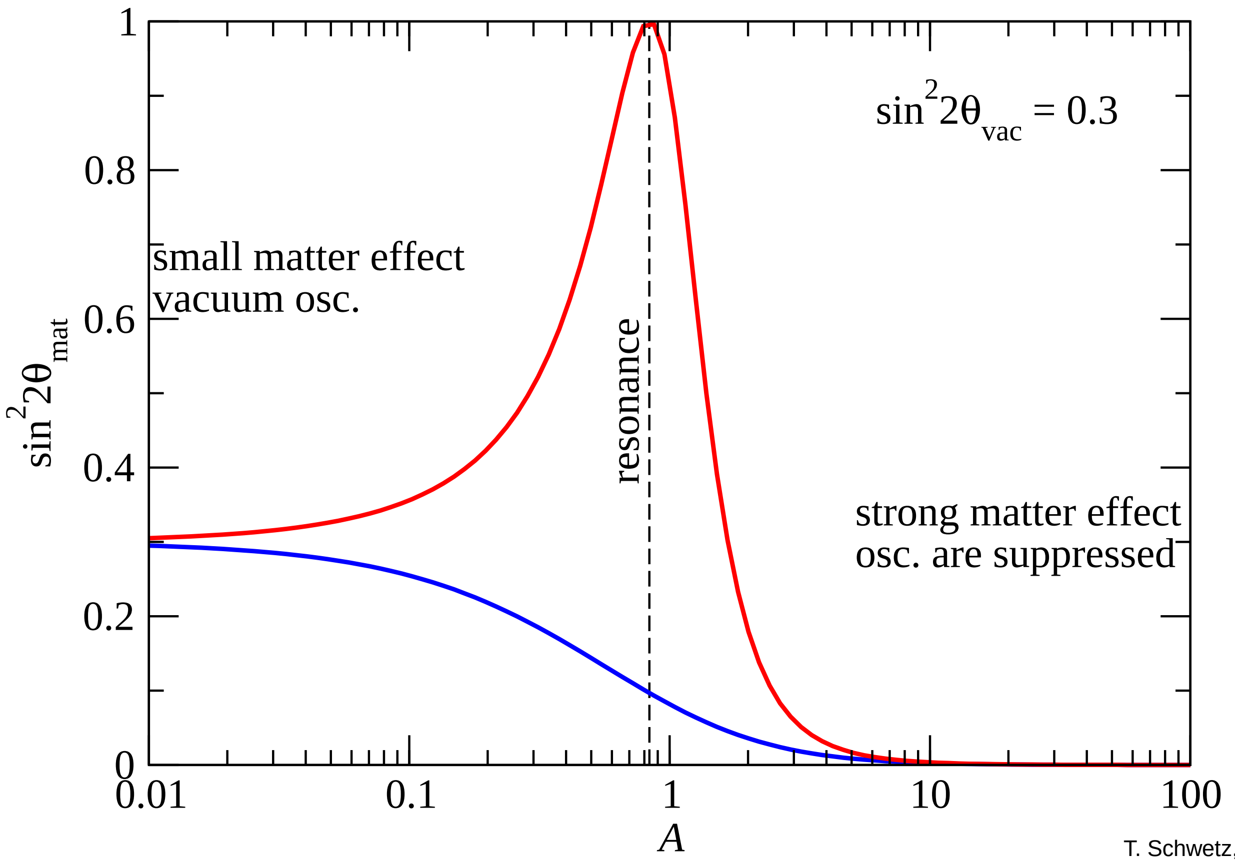
- introduction to the “neutrino hierarchy problem” in long-baseline experiments
- the regime of strong matter effect (baselines $\gtrsim$ 1000 km)
  comparison of ν and $\bar{\nu}$ oscillations
- the regime of small matter effect (baselines $\lesssim$ 1000 km)
  combination of all four oscillation chanels
  $\nu_\mu \to \nu_e\,, \quad \overline{\nu}_\mu \to \overline{\nu}_e\,, \quad \nu_e \to \nu_\mu\,, \quad \overline{\nu}_e \to \overline{\nu}_\mu$
- atmospheric neutrino data

The matter effect as solution for the hierarchy problem

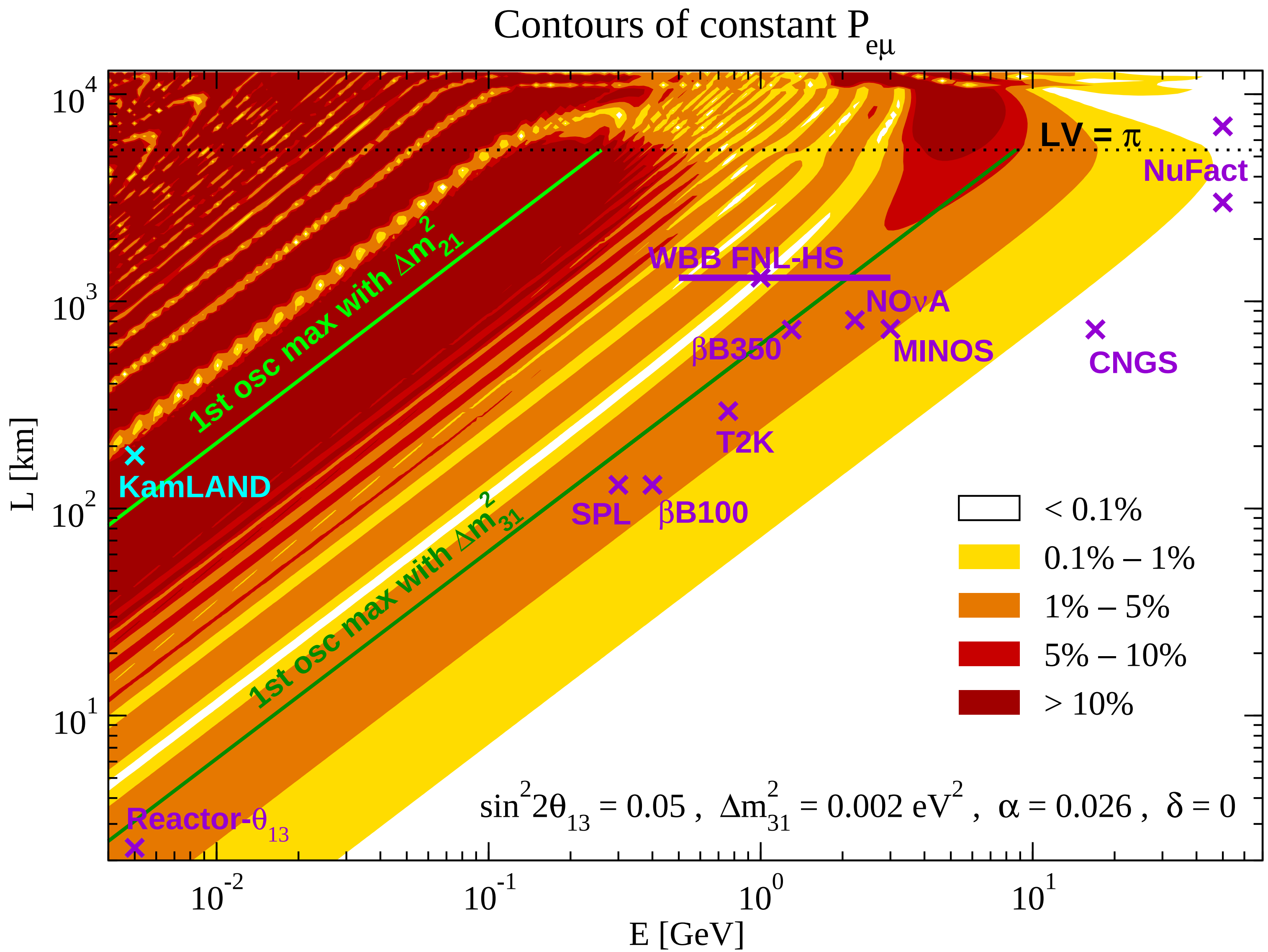
The MSW resonance (2ν)

$$\sin^2 2\theta_{\rm mat} = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \qquad A \equiv \pm \frac{2EV}{\Delta m^2}$$

resonance for $\cos 2\theta = A$



3ν LBL probability with earth matter profile



The "golden" formula

$$P_{\rm app} \approx 2\, s_{13}^2 \, \frac{\sin^2 \Delta(1-ahA)}{(1-ahA)^2} + \frac{1}{2}\,\frac{\tilde\alpha^2}{\Delta^2}\,\frac{\sin^2 A\Delta}{A^2}$$
$$+\, 2h\,\frac{\tilde\alpha}{\Delta}\, s_{13}\, \cos(h\Delta - at\delta_{\rm CP})\,\frac{\sin \Delta A}{A}\,\frac{\sin\Delta(1-ahA)}{1-ahA}$$

with $\theta_{23} = \pi/4$ and the definitions

$$\tilde\alpha \equiv \sin 2\theta_{12}\,\frac{\Delta m^2_{21} L}{4E}\,, \qquad \Delta \equiv \frac{|\Delta m^2_{31}| L}{4E}\,, \qquad A \equiv \left|\frac{2EV}{\Delta m^2_{31}}\right|$$

$$a = \begin{cases} +1 & \text{for } \nu \\ -1 & \text{for } \overline{\nu} \end{cases}, \quad t = \begin{cases} +1 & \text{for } e \to \mu \\ -1 & \text{for } \mu \to e \end{cases}, \quad h = \mathrm{sgn}(\Delta m^2_{31})$$

The size of the matter effect

$$A \simeq 0.09 \left(\frac{E}{\mathrm{GeV}}\right) \left(\frac{|\Delta m^2_{31}|}{2.5 \times 10^{-3}\,\mathrm{eV}^2}\right)^{-1}$$

for experiments at the 1st osc. max, $|\Delta m^2_{31}|L/2E \simeq \pi$, and

$$A \simeq 0.02 \left(\frac{L}{100\,\mathrm{km}}\right)$$

need $L \gtrsim 2000$ km and $E_\nu \gtrsim 5$ GeV in order to reach the regime of strong matter effect $A \gtrsim 0.5$.

The sign(Δm^2_{31}) degeneracy

The sign(Δm^2_{31}) degeneracy in vacuum

the "golden" formula in vacuum:

$$P^{\rm vac}_{\rm app} \approx 2\, s^2_{13}\, \sin^2\Delta + \frac{1}{2}\,\tilde\alpha^2 + 2h\,\tilde\alpha\, s_{13}\, \sin\Delta\cos(h\Delta - at\delta_{\rm CP})$$

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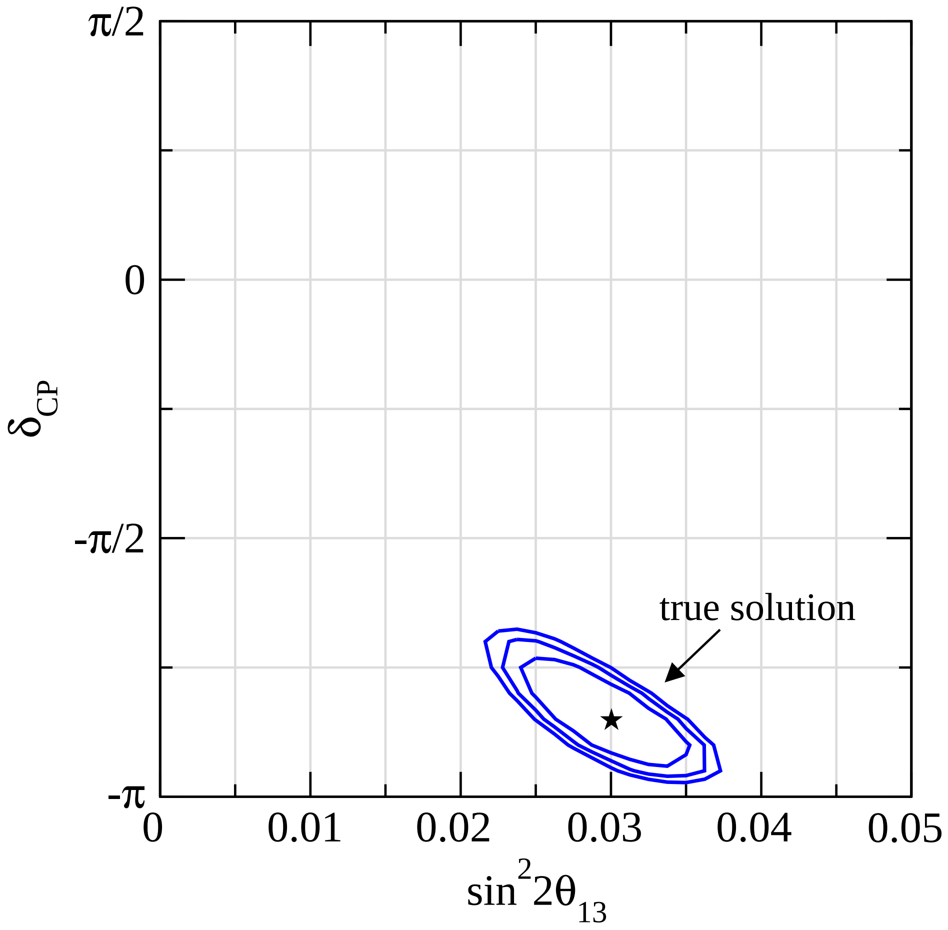
the degeneracy: Minakata, Nunokawa, hep-ph/0108085

$$P^{\rm vac}_{\nu_\mu\to\nu_e}(s_{13},\delta_{\rm CP},{\rm NH}) = P^{\rm vac}_{\nu_\mu\to\nu_e}(s_{13},\pi-\delta_{\rm CP},{\rm IH})$$
$$P^{\rm vac}_{\bar\nu_\mu\to\bar\nu_e}(s_{13},\delta_{\rm CP},{\rm NH}) = P^{\rm vac}_{\bar\nu_\mu\to\bar\nu_e}(s_{13},\pi-\delta_{\rm CP},{\rm IH})$$

(holds for all neutrino energies)

The sign(Δm^2_{31}) degeneracy for T2HK

Example from actual simulation of experiment: T2HK



True values:

$\sin^2 2\theta_{13} = 0.03$

$\delta_{\rm CP} = -0.85\pi$

$\sin^2 \theta_{23} = 0.4$

$\Delta m^2_{31} = 2.2 \times 10^{-3}\text{eV}^2$

spectral information resolves intrinsic deg.

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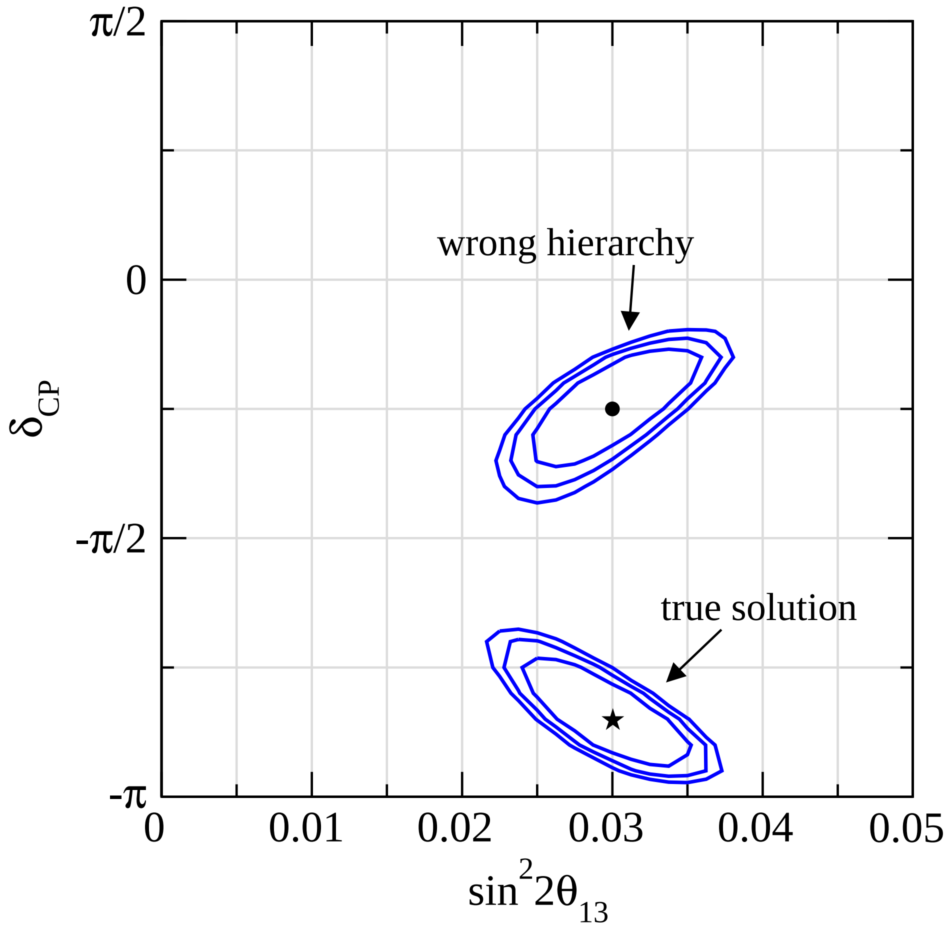

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The sign(Δm^2_{31}) deg. with small matter effect

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$$P_{\nu_\mu\to\nu_e}(s_{13}, \delta_{\rm CP}, {\rm NH}) = P_{\nu_\mu\to\nu_e}(s'_{13}, \delta'_{\rm CP}, {\rm IH})$$

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small deviation from vacuum solution:

$$s'_{13} = s_{13}(1+\epsilon_s)\,, \qquad \delta'_{\rm CP} = \pi - \delta_{\rm CP} + \epsilon_\delta$$

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$\Rightarrow$ two linear equations for ϵ_s and ϵ_δ

The sign(Δm^2_{31}) deg. with small matter effect

$$\epsilon_s(2s_{13} + at\,\tilde\alpha\,\sin\delta_{\rm CP}) - at\,\epsilon_\delta\,\tilde\alpha\,\cos\delta_{\rm CP}$$
$$= 2A(2ahs_{13} + ht\,\tilde\alpha\,\sin\delta_{\rm CP})$$

has a unique solution for $a = \pm 1$ (ν and $\overline{\nu}$):

$$\epsilon_s = ht\,A\,\frac{\tilde\alpha}{s_{13}}\,\sin\delta_{\rm CP}$$
$$\epsilon_\delta = ht\,\frac{A}{\cos\delta_{\rm CP}}\left(\frac{\tilde\alpha}{s_{13}}\,\sin^2\delta_{\rm CP} - 4\frac{s_{13}}{\tilde\alpha}\right)$$

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The leading order matter effect cannot break the degeneracy! (need $A \sim 1$ to explore non-linear effects)

The regime of strong matter effect

The regime of strong matter effect

$L \gtrsim$ 1000 km and $E_\nu \gtrsim$ few GeV

- US wide band beam (FNL to DUSEL, 1300 km)
- T2KK (detector for T2K beam in Korea, 1050 km)
- Neutrino factory (3000 km, 7000 km)

Strong matter effects

Look for the matter resonance:

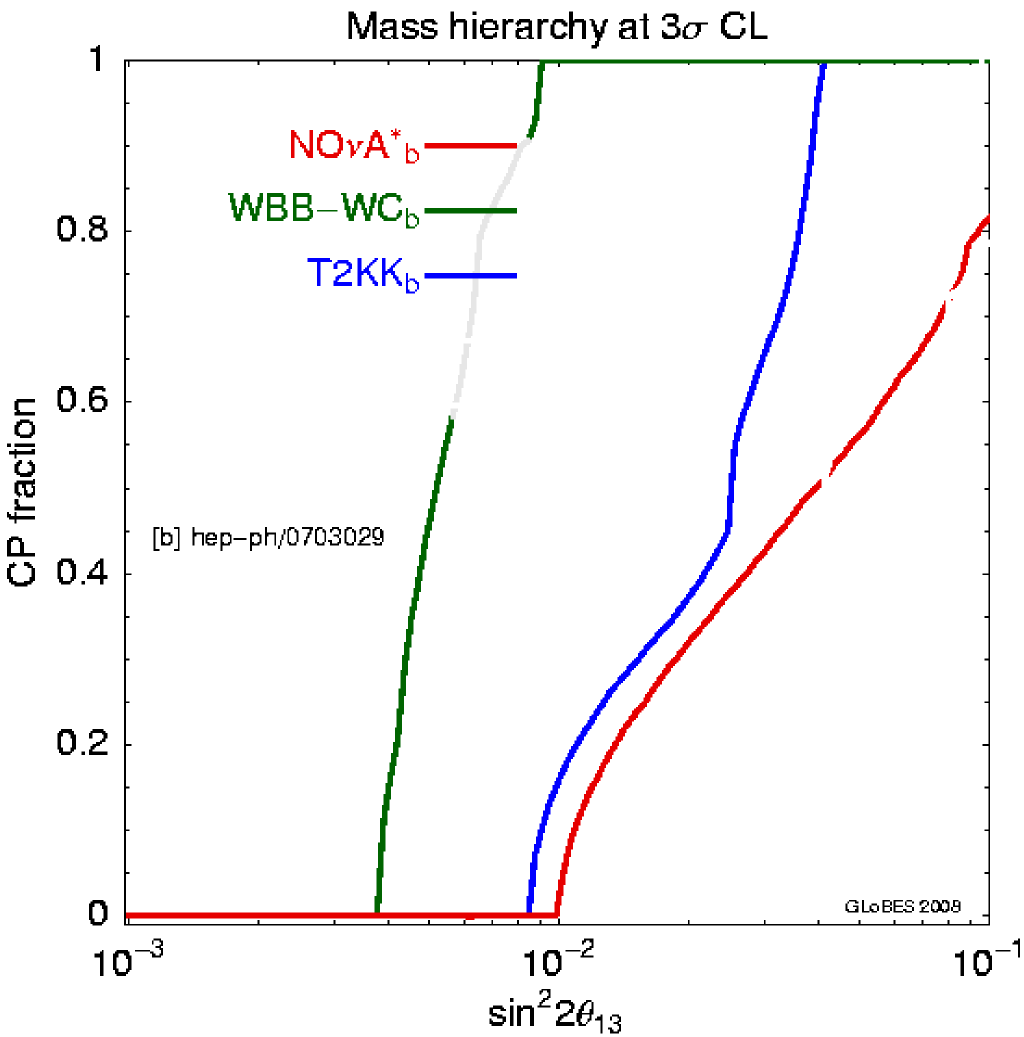
$$ahA \equiv ah\left|\frac{2EV}{\Delta m^2_{31}}\right| = \cos 2\theta_{13} \approx 1$$

for resonance need $ah = 1$

Find out whether the resonance occurs
for neutrinos ($a = +1$) or
for anti-neutrinos ($a = -1$)

$\Rightarrow$ will tell you the hierarchy ($h = +1$ or -1)

SB mass hierarchy sensitivities



NOνA*:
100 kt LAr @ 820 km
3 yr ν, 3 yr $\bar{\nu}$ @ 1.1 MW

T2KK:
270 kt WC @ 295 & 1050 km
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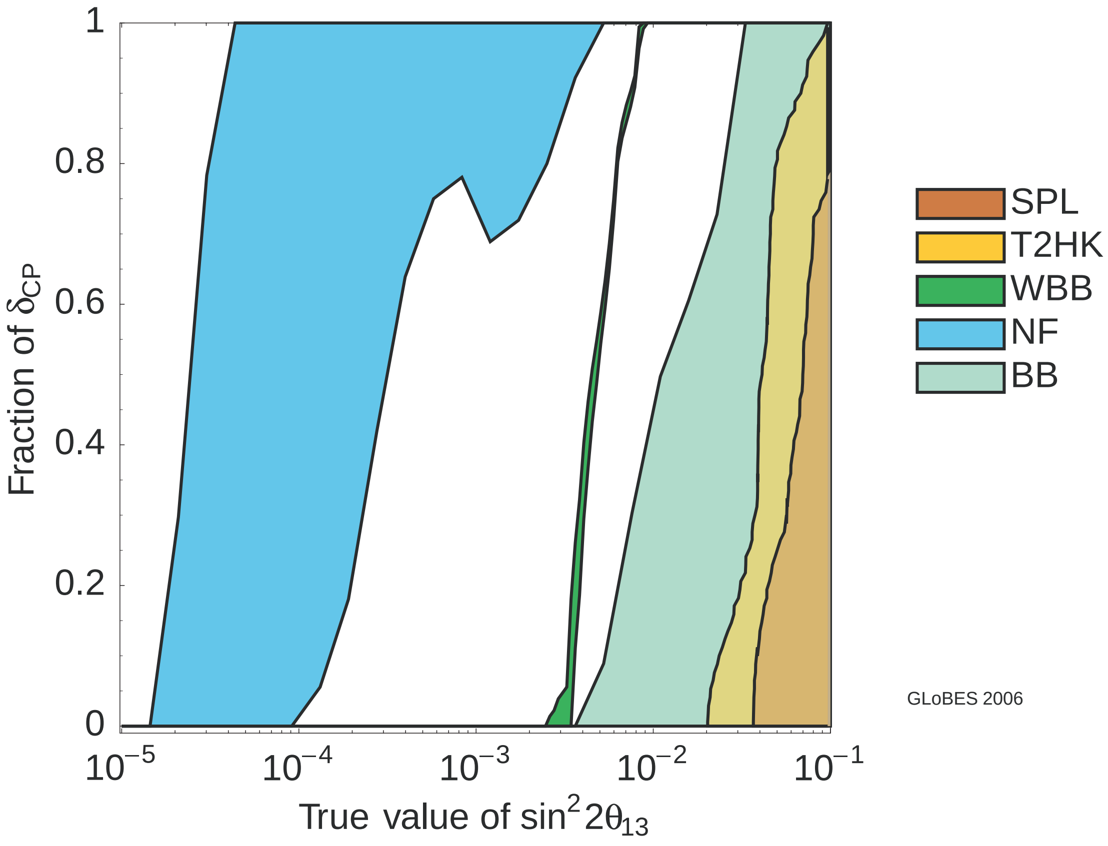
WBB:
300 kt WC @ 1290 km
5yr ν @ 1 MW, 5yr $\bar{\nu}$ @ 2 MW

Barger, Huber, Marfatia, Winter, hep-ph/0703029

ISS mass hierarchy sensitivities

ultimate sensitivity at Neutrino Factories
($L = 3000/7000$ km)



ISS Physics Working Group report, arxiv:0710.4947

The regime of small matter effect

The sign(Δm^2_{31}) deg. with small matter effect

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with $s'_{13} = s_{13}(1+\epsilon_s)\,, \qquad \delta'_{\rm CP} = \pi - \delta_{\rm CP} + \epsilon_\delta$

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$\Rightarrow$ the degeneracy is broken if all four channels

$$\nu_\mu \longrightarrow \nu_e\,, \quad \overline{\nu}_\mu \longrightarrow \overline{\nu}_e\,, \quad \nu_e \longrightarrow \nu_\mu\,, \quad \overline{\nu}_e \longrightarrow \overline{\nu}_\mu$$

are available TS, hep-ph/0703279

The sign(Δm^2_{31}) deg. with small matter effect

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the signs of the solutions for $a = \pm 1$ depend on t:

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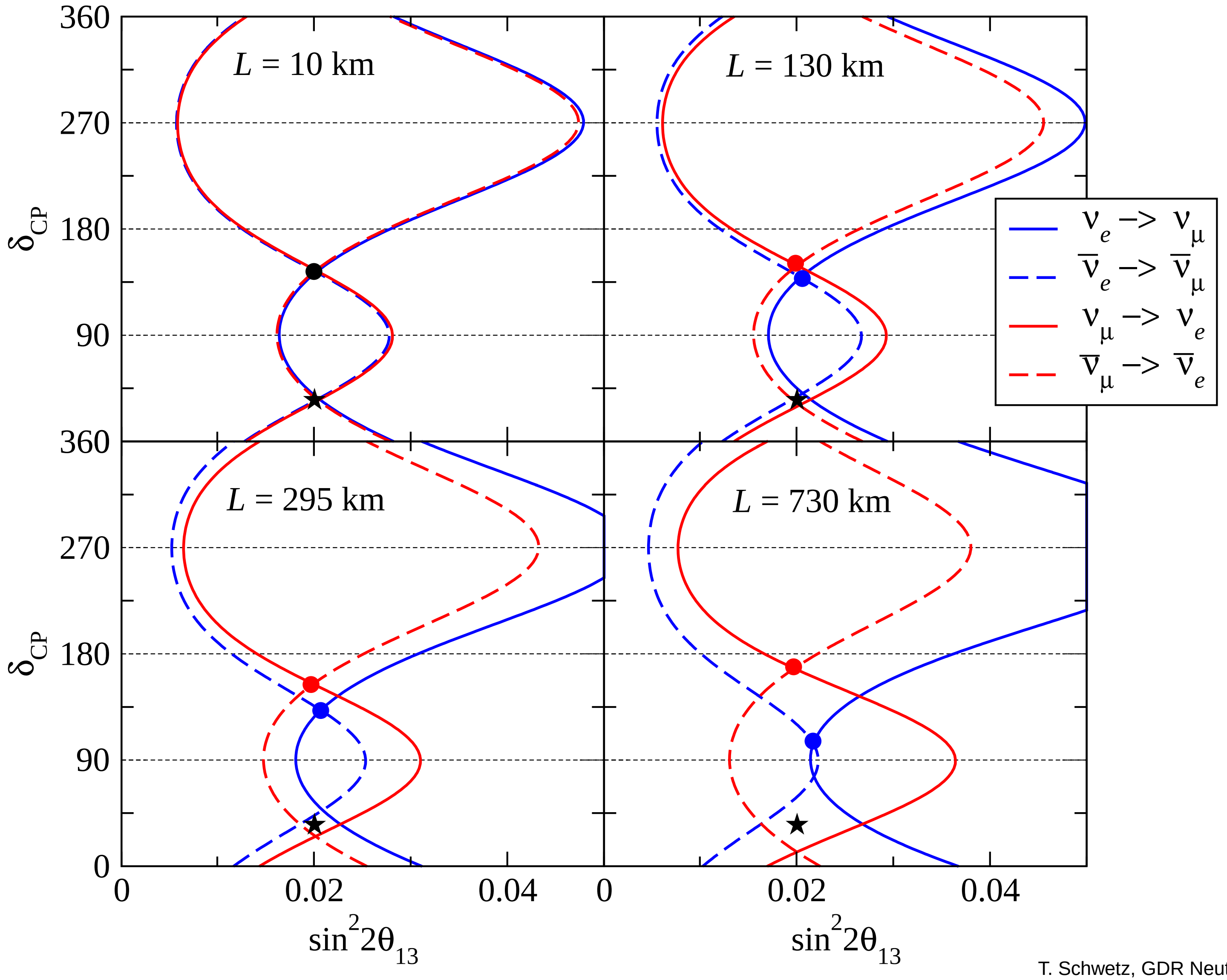
$\Rightarrow$ the degenerate solutions move in opposite directions for $\mu \to e$ and $e \to \mu$

Location of the degeneracy

equal probability contours for the wrong hierarchy:



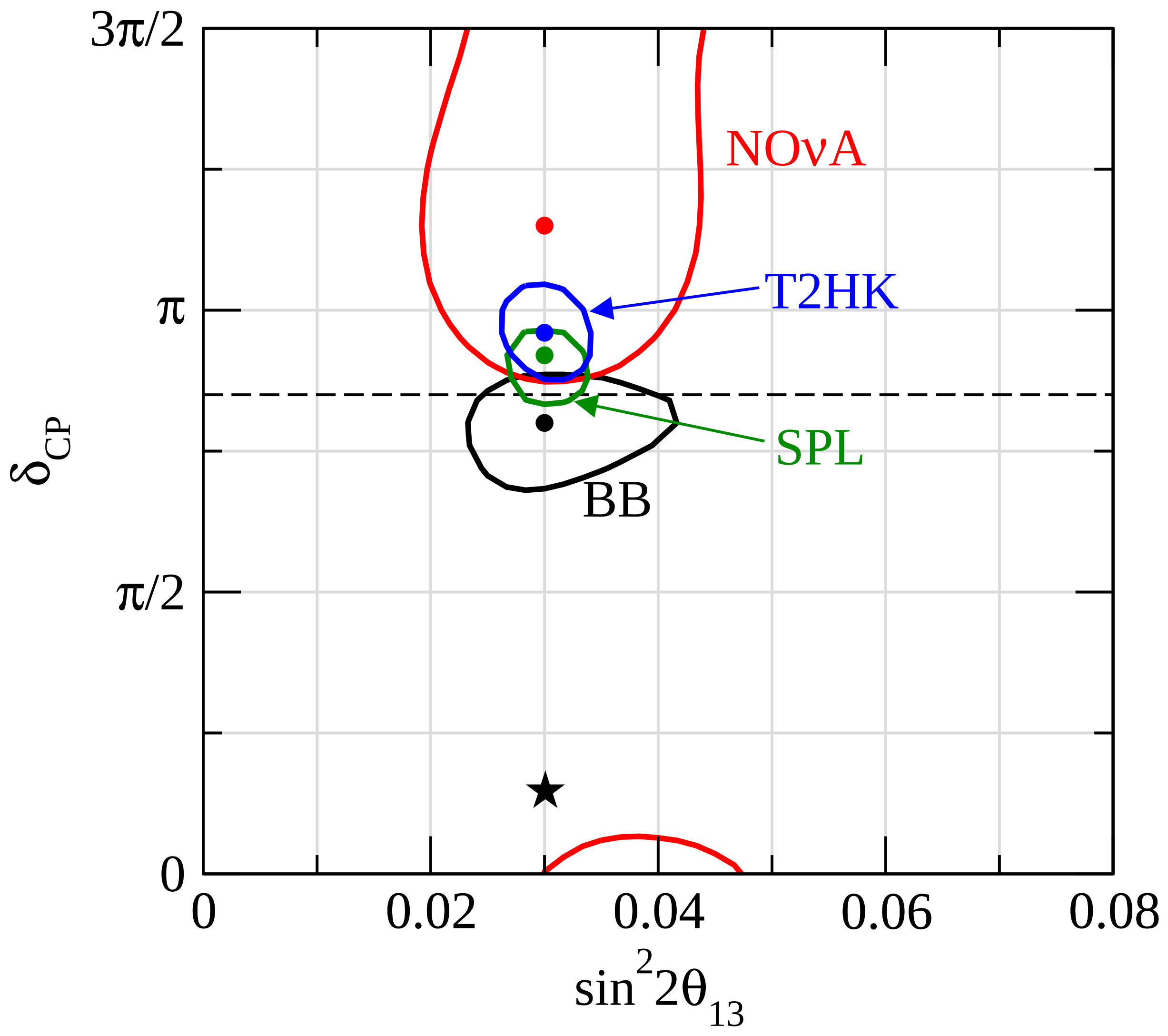
Does it work in “real life”?

Example setups

pre-defined setups taken from GLoBES:

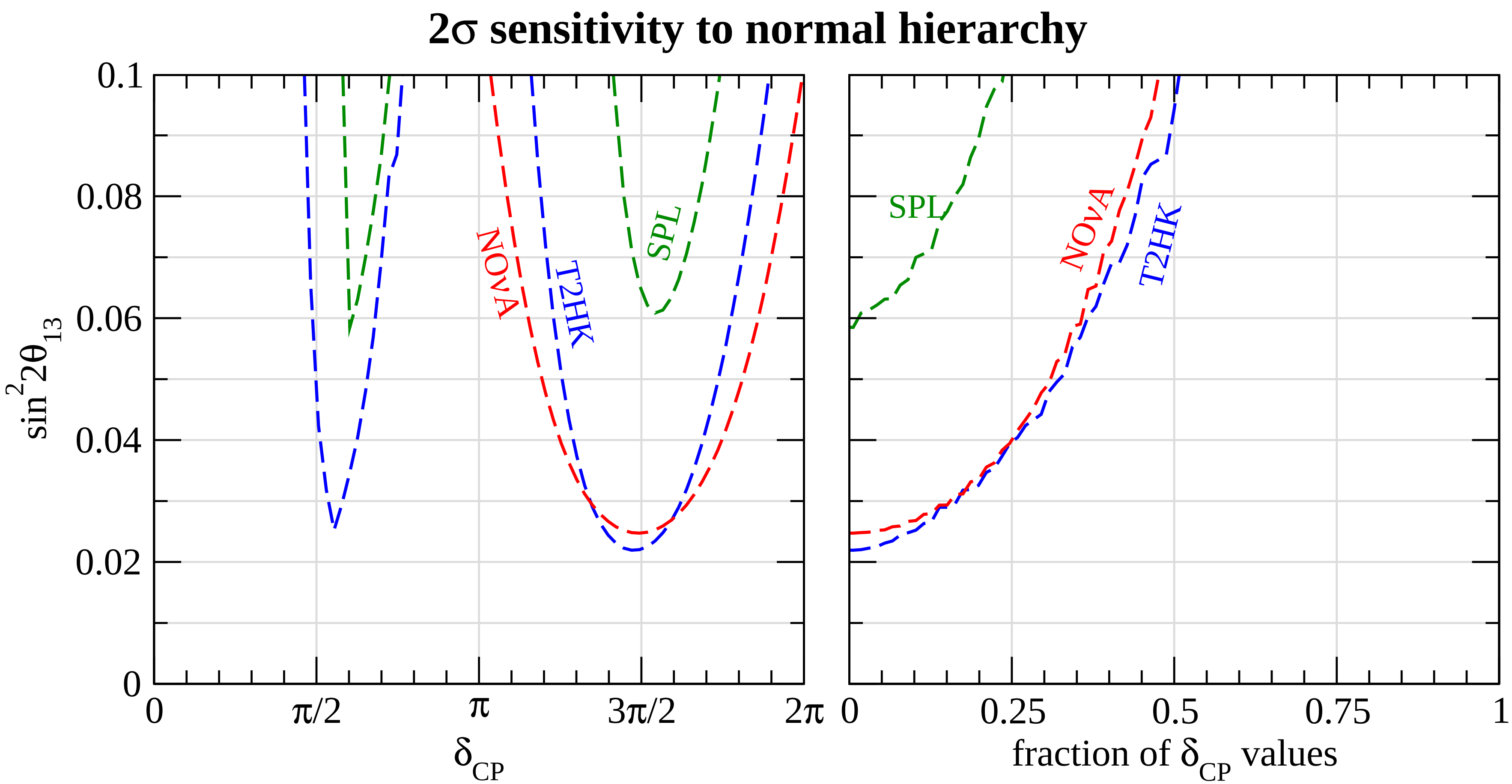
Exp.	$\frac{L}{\text{km}}$	$\frac{\langle E \rangle}{\text{GeV}}$	Detector	$\frac{T}{\text{yr}}$	Beam	σ_{sys}
BB100	130	0.4	500 kt WC	$4\nu + 4\overline{\nu}$	2.2/5.8e18	2%
SPL	130	0.3	500 kt WC	$2\nu + 8\overline{\nu}$	4 MW	2%
T2HK	295	0.8	500 kt WC	$4\nu + 4\overline{\nu}$	4 MW	5%
NOνA	812	2.0	25 kt TASD	$3\nu + 3\overline{\nu}$	1.12 MW	5%

Location of the degeneracy



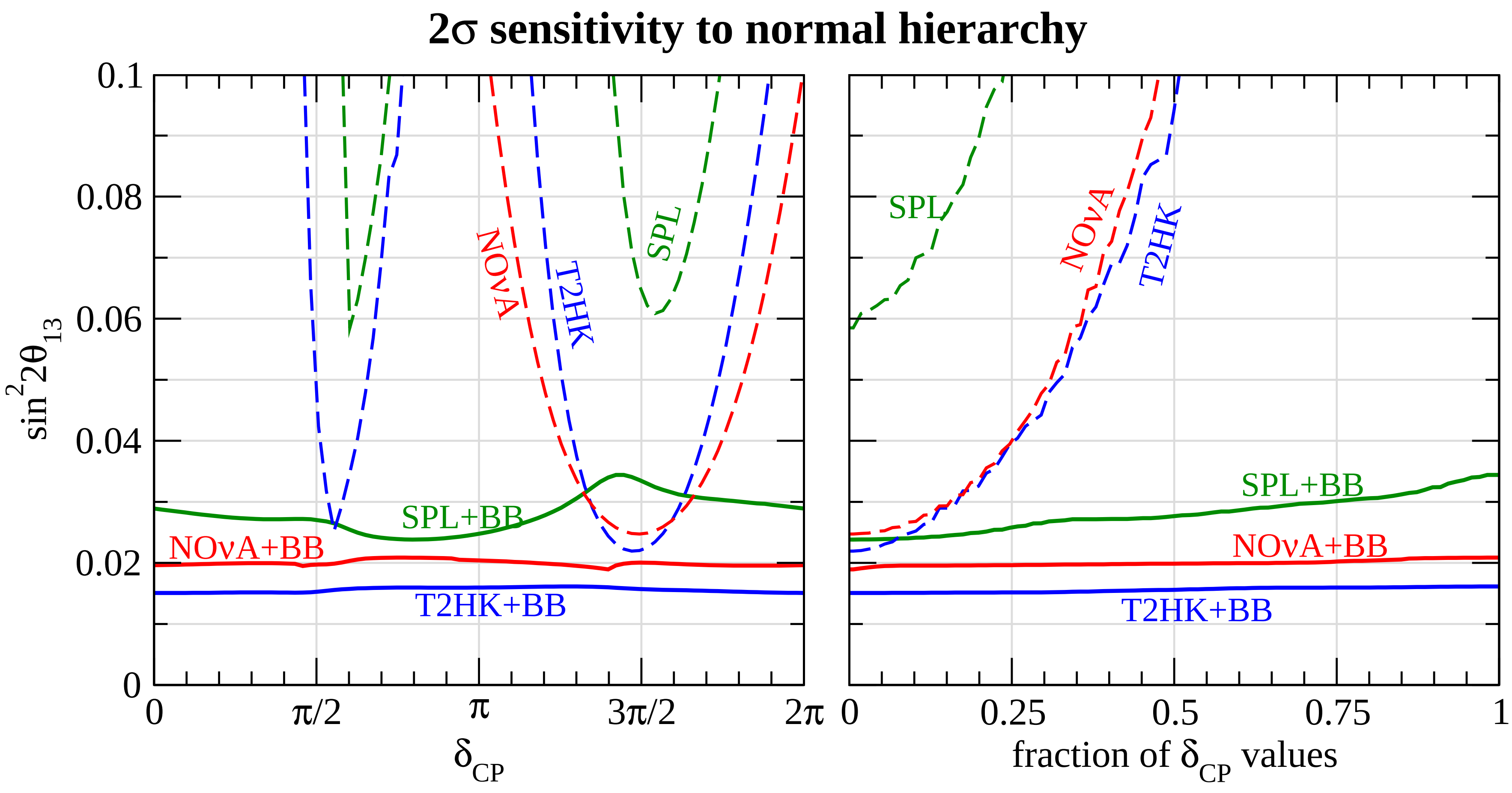
Sensitivity SB+BB100

Superbeams $(\mu \to e)$



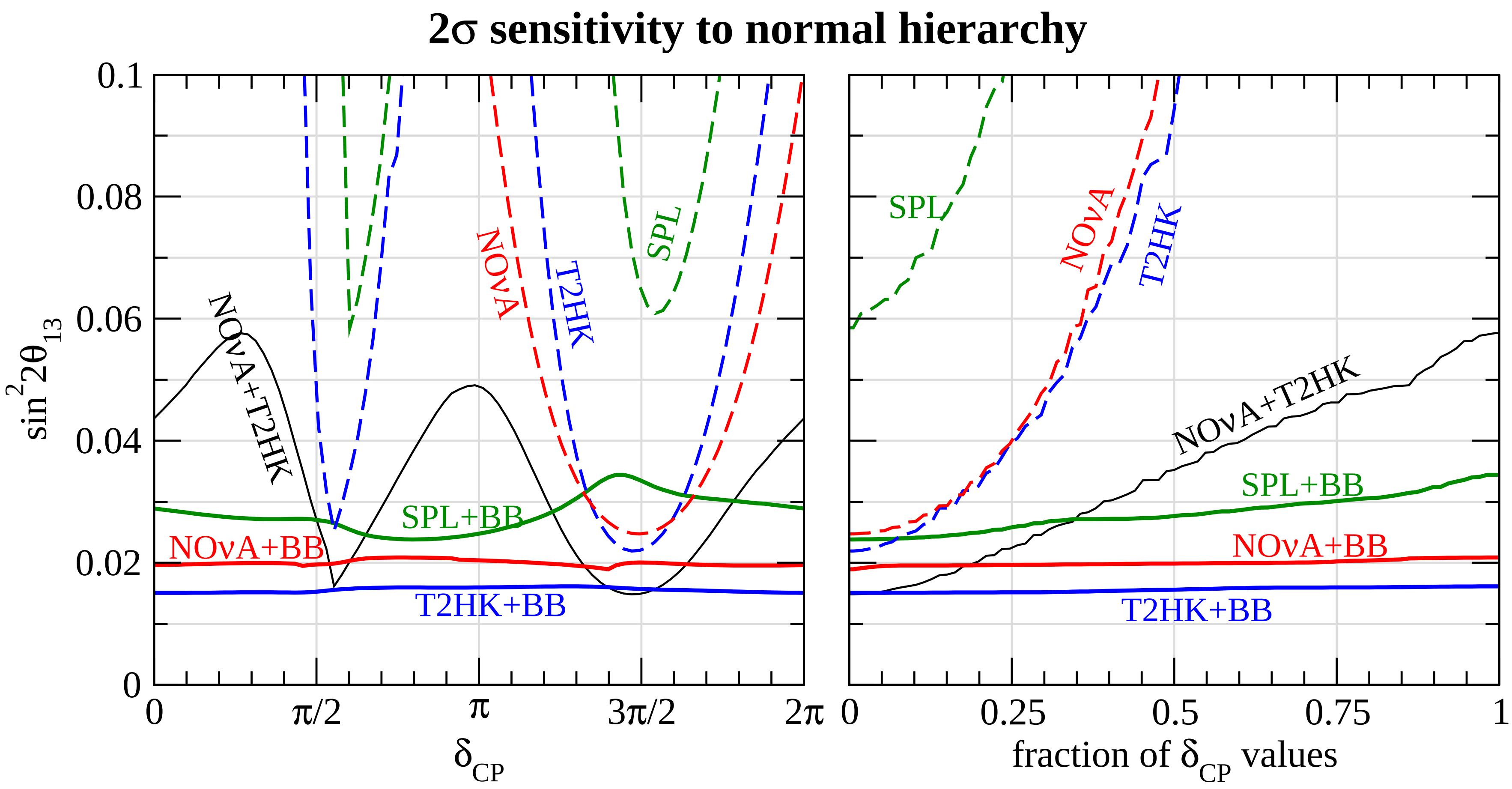
Sensitivity SB+BB100

Superbeams $(\mu \to e)$ combined with BB100 $(e \to \mu)$



Sensitivity SB+BB100

Superbeams $(\mu \rightarrow e)$ combined with BB100 $(e \rightarrow \mu)$



Side remark: another synergy of

$\nu_\mu \longrightarrow \nu_e$ and $\nu_e \longrightarrow \nu_\mu$ channels

Replacing anti-neutrino running

CPT invariance:

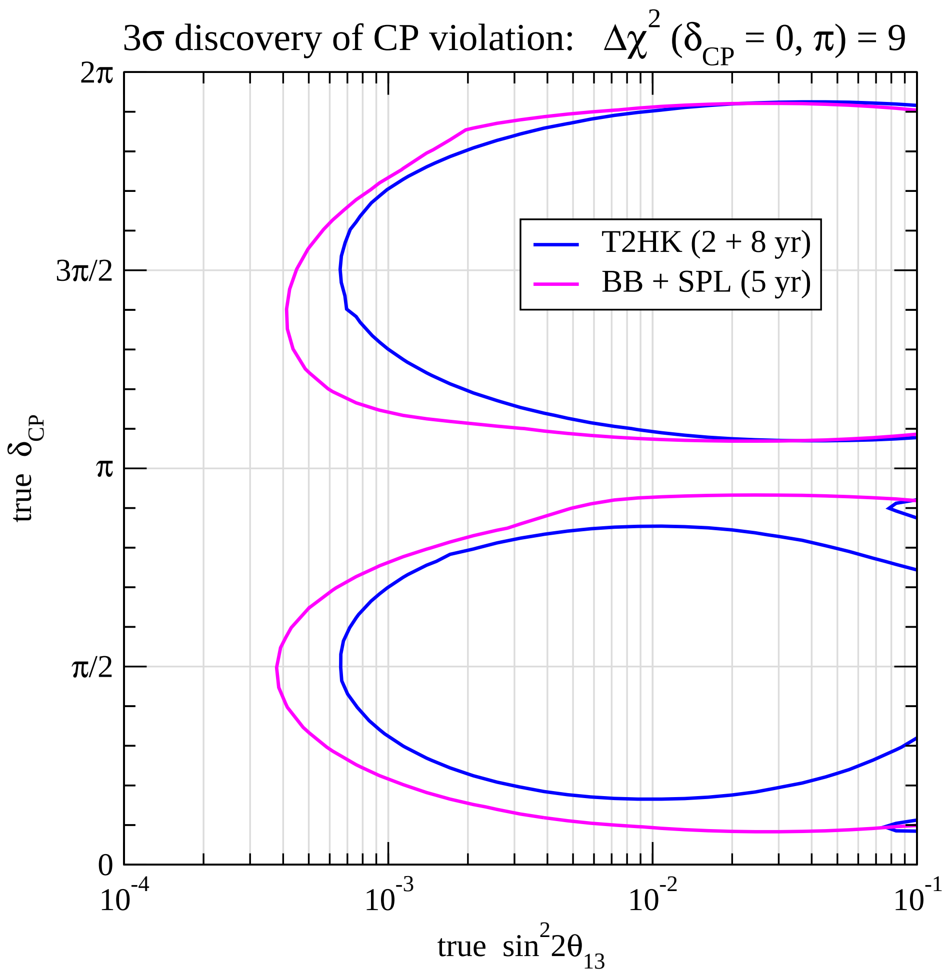
$$P_{\bar{\nu}_\mu \to \bar{\nu}_e} = P_{\nu_e \to \nu_\mu} + \mathcal{O}(A)$$

$\Rightarrow$ replace the anti-neutrinos from the superbeam with neutrinos from the beta beam

$\Rightarrow$ if beta beam and superbeam are available simultaneously anti-neutrino running is not needed for the θ_{13} and CPV measurements

$\Rightarrow$ can do the same measurement in about half of the time

BB+SPL (ν only): CP violation



Campagne, Maltoni, Mezzetto, Schwetz, hep-ph/0603172

Using atmospheric neutrinos

The hierarchy and atmospheric neutrinos

Explore 3-flavour effects in atmospheric neutrinos

- Water Cerenkov:
  - sees only sum $\nu + \bar{\nu} \longrightarrow$ dilution of the effect
  + can be made very big (Mt scale)
- Magnetized iron:
  + can distinguish ν from $\bar{\nu}$ events
  - electron detection difficult $\longrightarrow$ μ-like events

Megaton water Cerenkov

- Many proposed long-baseline experiments rely on a Mt-scale water Cerenkov detector

  WBB (UNO), CERN BB/SPL (MEMPHYS), T2K (HK)

- high statistics atmospheric neutrino data come “for free”

- combine LBL and atmospheric data

  Huber, Maltoni, Schwetz, hep-ph/0501037

3-flavour effects in atmospheric neutrinos

excess of electron-like events:

$$\frac{N_e}{N_e^0} - 1 \simeq \quad (r\, s_{23}^2 - 1)\, P_{2\nu}(\Delta m_{31}^2, \theta_{13}) \qquad \theta_{13}\text{-effects}$$

$$+ \quad (r\, c_{23}^2 - 1)\, P_{2\nu}(\Delta m_{21}^2, \theta_{12}) \qquad \Delta m_{21}^2\text{-effects}$$

$$- \quad 2 s_{13} s_{23} c_{23}\, r\, \mathrm{Re}(A_{ee}^* A_{\mu e}) \qquad \text{interference: } \delta_{\rm CP}$$

$$r = r(E_\nu) \equiv \frac{F_\mu^0(E_\nu)}{F_e^0(E_\nu)}$$

$r \approx 2$ (sub-GeV)

$r \approx 2.6 - 4.5$ (multi-GeV)

θ_{13}-effects

$$\frac{N_e}{N_e^0} - 1 \simeq (r\, s_{23}^2 - 1)\, P_{2\nu}(\Delta m_{31}^2, \theta_{13})$$

resonant matter effect in $P_{2\nu}(\Delta m_{31}^2, \theta_{13})$
for multi-GeV events ($r \approx 2.6 - 4.5$)

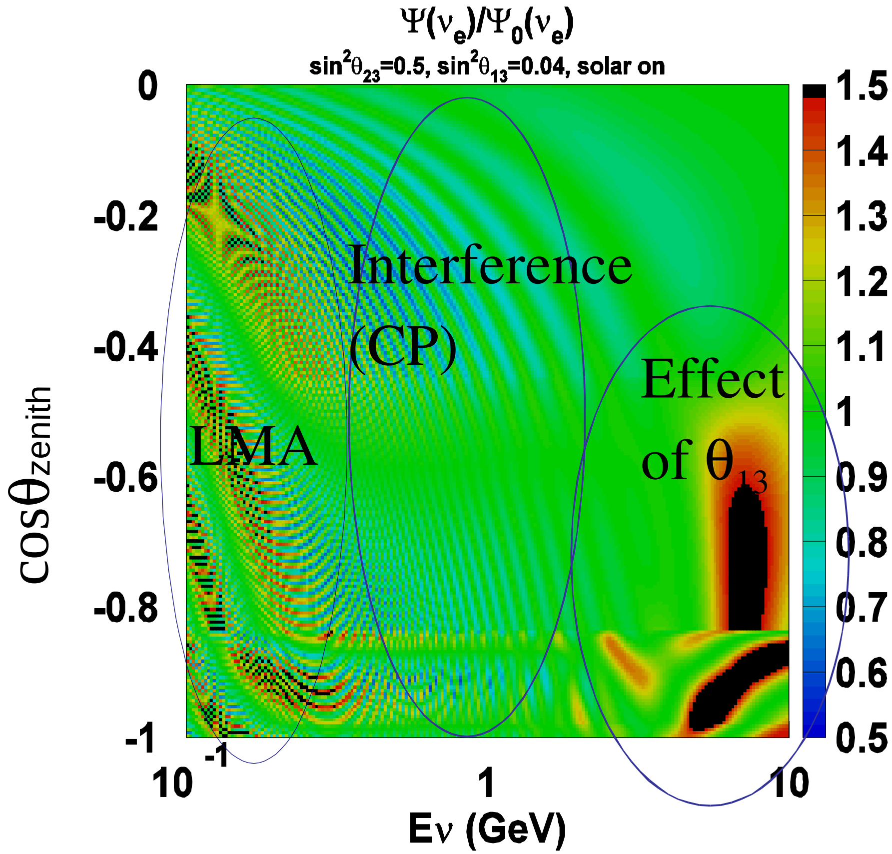
normal hierarchy: enhancement for neutrinos
inverted hierarchy: enhancement for anti-neutrinos

detection cross sections are different for neutrinos and anti-neutrinos

sensitivity to the neutrino mass hierarchy

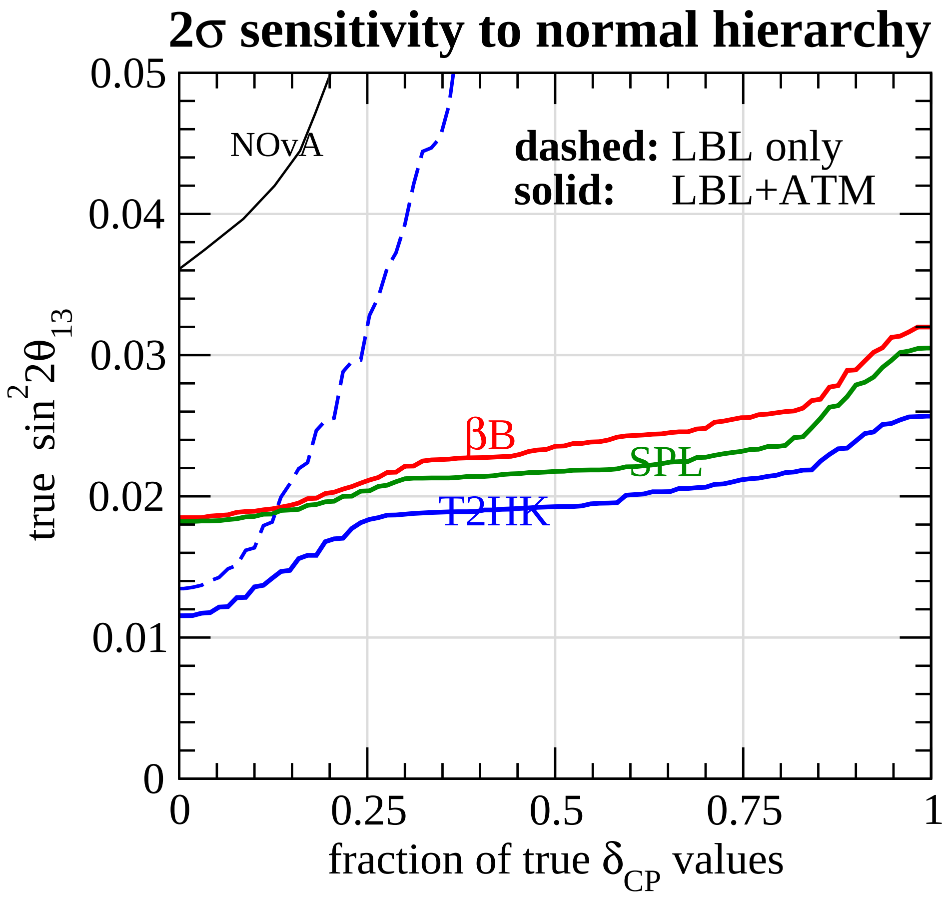
3-flavour effects in atmospheric neutrinos

$s^2\theta_{23}$=0.4
$s^2\theta_{13}$=0.04
$\delta_{cp}=\pi/4$



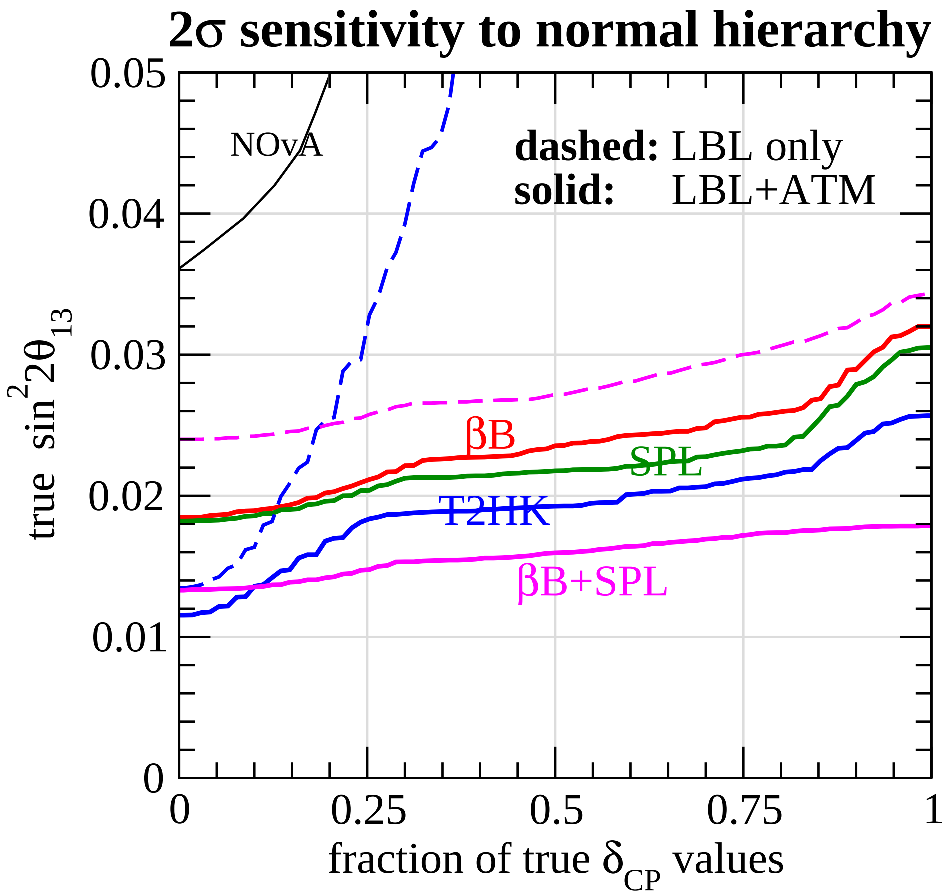
plot from T. Kajita

Hierarchy with LBL + ATM



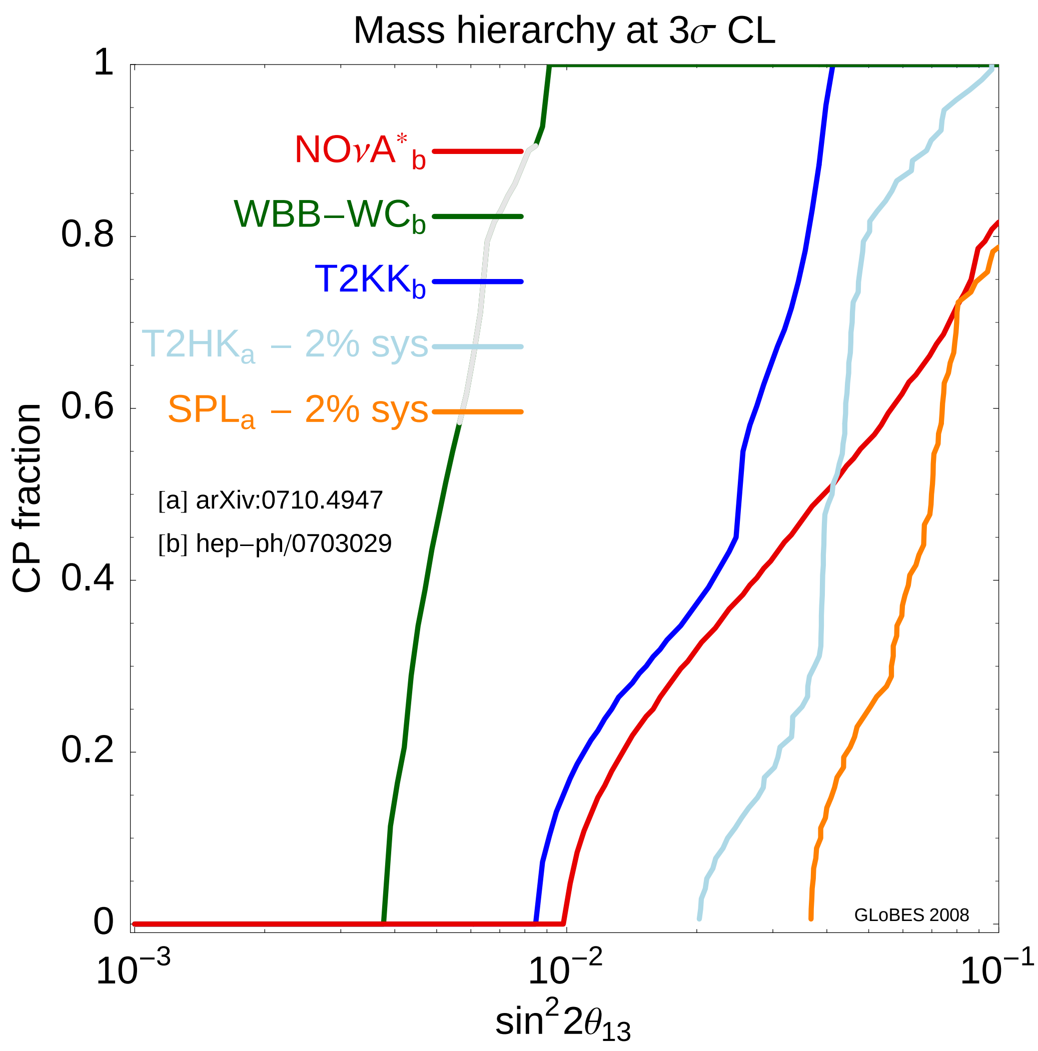
Campagne, Maltoni, Mezzetto, Schwetz, hep-ph/0603172

Hierarchy with LBL + ATM



Campagne, Maltoni, Mezzetto, Schwetz, hep-ph/0603172

Comparison w "strong matter effect" exps.



SPL (130 km) and T2HK (295 km) include 5 Mt yr WC atm neutrino data

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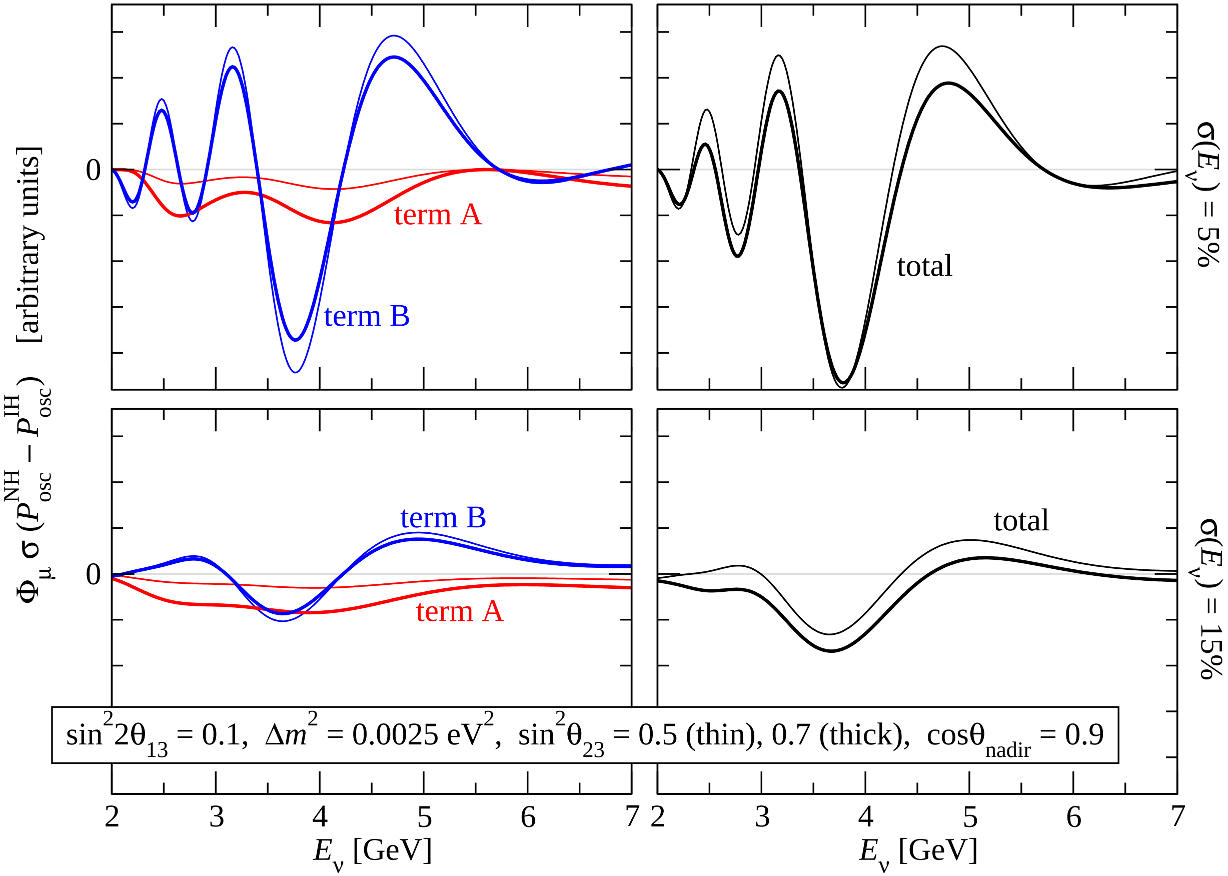
The hierarchy and magnetized detectors

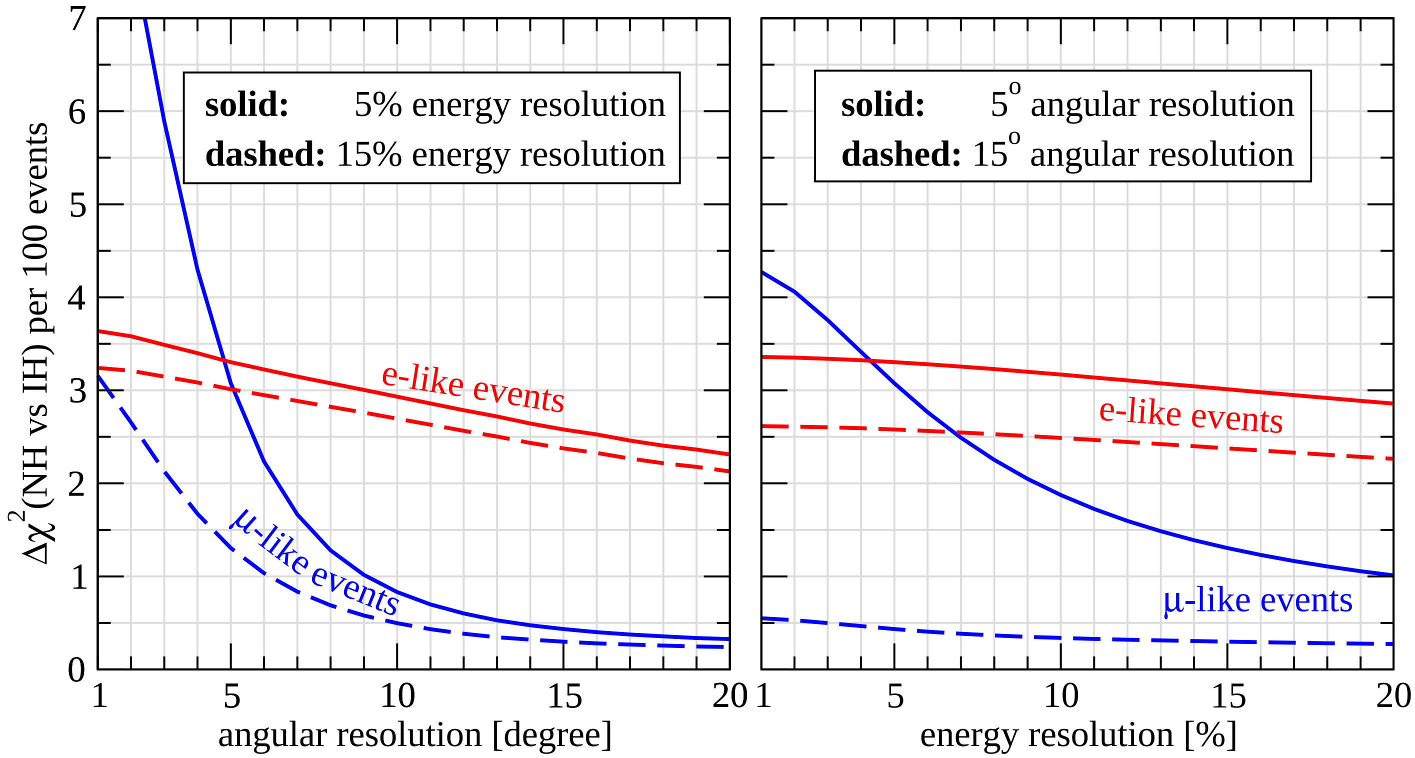
difference of the μ-like event spectra for NH and IH



$$\Delta S_\mu \propto \underbrace{\Phi_\mu \sigma \sin^2\theta_{23}\left(\frac{1}{r} - \sin^2\theta_{23}\right)\Delta P_{2\nu}}_{\text{term A}} + \underbrace{\Phi_\mu \sigma \frac{1}{2}\sin^2 2\theta_{23} \Delta \mathrm{Re}\,(A'_{33})}_{\text{term B}}$$

The hierarchy and magnetized detectors

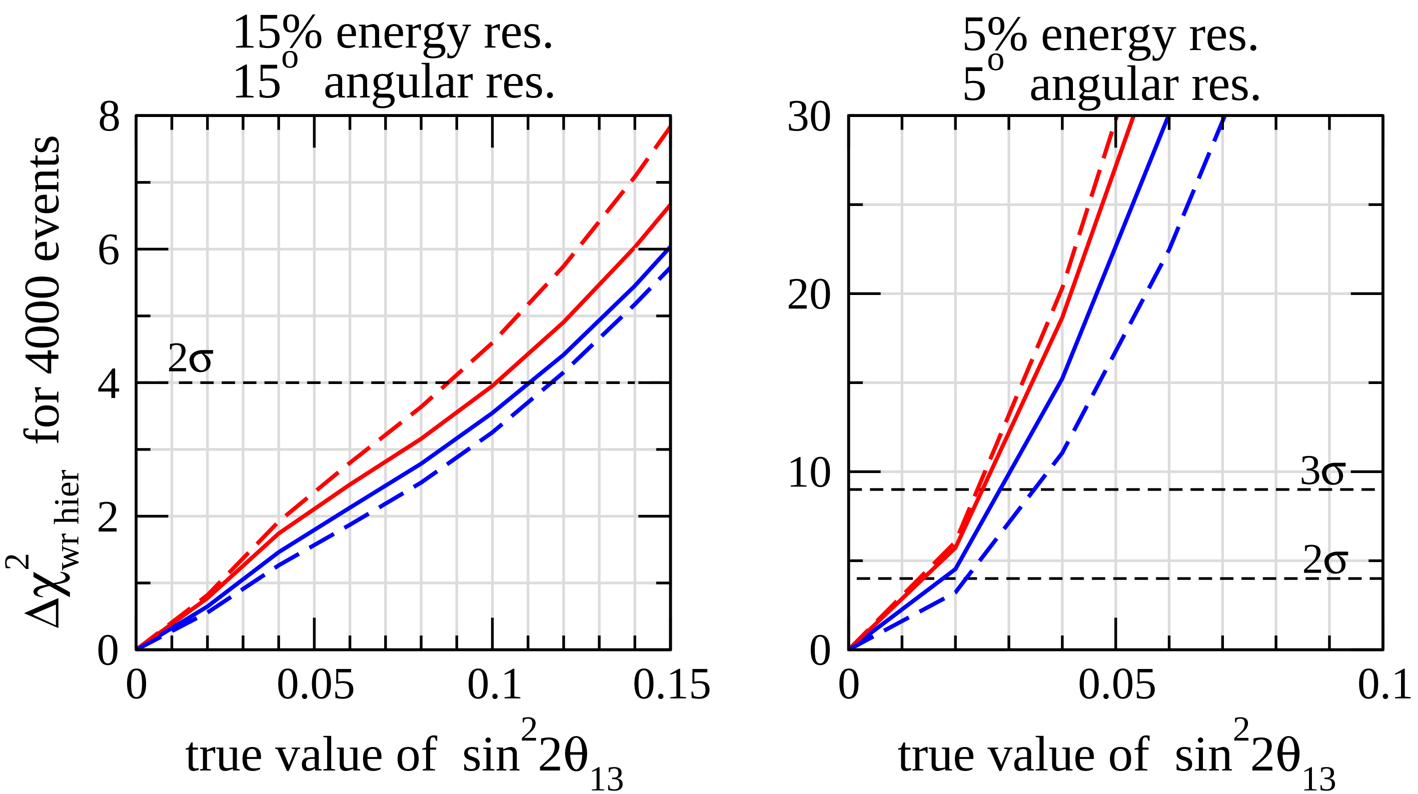
the ability to reconstruct the neutrino energy and neutrino direction is crucial



Petcov, Schwetz, hep-ph/0511277; Indumathi, Murthy, hep-ph/0407336

The hierarchy and magnetized detectors

consider 500 kty data (e.g., INO with 50 kt for 10 yrs)



osc. params. fixed, external prior information

solid: true hierarchy normal, **dashed:** true hierarchy inverted

Concluding remarks

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  - combine $\mu \to e$ and $e \to \mu$ channels (2 exps!)
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Thank you for your attention