Sensitivity of CNGS and J-PARC Beams to Quantum-Gravity Decoherence in Neutrino Oscillations

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Quantum decoherence

The time evolution of a closed quantum system

$$\frac{\partial \rho}{\partial t} = L[\rho] = -i[H, \rho] \qquad \rho > 0$$

Pure state $Tr(\rho^2) = Tr\rho = 1$

Mixed state $Tr(\rho^2) < Tr\rho = 1$

Space-time foam environment

$$\frac{\partial \rho}{\partial t} = \Lambda_1 \rho + \Lambda_2 \rho$$

Nonunitary evolution (Lindblad mapping)

H with stochastic element in a classical metric

Generically space-time foam and the back-reaction of matter on the gravitational metric may be modeled as a randomly fluctuating environment

MSW-like effect

$$H_{\mathrm{eff}} = H + n^{\mathrm{c}}_{\mathrm{bh}}(r) H_I \qquad H_I = \begin{pmatrix} a_{\nu_\mu} & 0 \\ 0 & a_{\nu_\tau} \end{pmatrix}$$

Foam medium is assumed to be to be described by Gaussian random variable

$\langle n^c_{\mathrm{bh}}(t) \rangle = n_0$ The average number of foam particles

$$\langle n^c_{bh}(t) n^c_{bh}(t') \rangle \sim \Omega^2 n_0^2 \delta(t - t')$$

The modified time evolution (master equation)

$$\frac{\partial}{\partial t}\langle \rho \rangle = -i[H + n_0 H_I, \langle \rho \rangle] - \Omega^2 n_0^2 [H_I, [H_I, \langle \rho \rangle]]$$

$$\langle \rho \rangle^{(\nu_\mu)} = \frac{1}{2} 1_2 + \sin(2\theta)\frac{s_1}{2} + \cos(2\theta)\frac{s_3}{2}$$

$$\langle \rho \rangle^{(\nu_\tau)} = \frac{1}{2} 1_2 - \sin(2\theta)\frac{s_1}{2} - \cos(2\theta)\frac{s_3}{2}$$

$$P_{\nu_\mu \to \nu_\tau}(t) = Tr\left(\langle \rho \rangle (t) \langle \rho \rangle^{(\nu_\tau)} \right)$$

$$P_{\nu_\mu \to \nu_\tau} =$$

$$\frac{1}{2} + e^{-\Delta a_{\mu\tau}^2 \Omega^2 t (1 + \frac{\Delta_{12}^2}{4\Gamma}(\cos(4\theta)-1))} \sin(t\sqrt{\Gamma}) \sin^2(2\theta) \Delta a_{\mu\tau}^2 \Omega^2 \Delta_{12}^2 \left(\frac{3\sin^2(2\theta)\Delta_{12}^2}{4\Gamma^{5/2}} - \frac{1}{\Gamma^{3/2}} \right)$$

$$- e^{-\Delta a_{\mu\tau}^2 \Omega^2 t (1 + \frac{\Delta_{12}^2}{4\Gamma}(\cos(4\theta)-1))} \cos(t\sqrt{\Gamma}) \sin^2(2\theta) \frac{\Delta_{12}^2}{2\Gamma}$$

$$- e^{-\frac{\Delta a_{\mu\tau}^2 \Omega^2 t \Delta_{12}^2 \sin^2(2\theta)}{\Gamma}} \frac{(\Delta a_{\mu\tau} + \cos(2\theta)\Delta_{12})^2}{2\Gamma}$$

$$\Gamma = (\Delta a_{\mu\tau}\cos(2\theta) + \Delta_{12})^2 + \Delta a_{\mu\tau}^2 \sin^2(2\theta) \quad \Delta_{12} = \frac{\Delta m_{12}^2}{2k} \qquad \Delta a_{\mu\tau} \equiv a_{\nu_\mu} - a_{\nu_\tau}$$

Damping exponent

$$\text{exponent} \sim -\Delta a_{\mu\tau}^2 \Omega^2 t f(\theta) \;;\; f(\theta) = 1 + \frac{\Delta_{12}^2}{4\Gamma}(\cos(4\theta) - 1) \;,\; \text{or} \; \frac{\Delta_{12}^2 \sin^2(2\theta)}{\Gamma}$$

$$\Delta a_{\mu\tau} \propto G_N n_0$$

Stochastic fluctuations of Space-Time metric backgrounds

1+1, no spin

$$g = \mathcal{O}\eta\mathcal{O}^T \qquad \eta = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \mathcal{O} = \begin{pmatrix} a_1 + 1 & a_2 \\ a_3 & a_4 + 1 \end{pmatrix}$$

$\langle a_i \rangle = 0 \qquad \langle a_i a_j \rangle = \delta_{ij}\sigma_i$ - random variables

$\phi(x,t) \simeq \varphi(k,\omega)e^{-i(-\omega t + kx)}$ - plane wave solution of KG equation

$\omega = \omega(g^{\mu\nu}, k, M)$ - dispersion relation

$$\text{Prob}(1 \to 2) = \sum_{j,l} U_{1j}U_{2j}^* U_{1l}^* U_{2l} e^{i(\omega_l - \omega_j)t}$$

$$P_{\nu_\mu \to \nu_\tau}(t) = U_{12}U_{22}^* U_{11}^* U_{21} e^{i(\omega_1 - \omega_2)t} + U_{11}U_{21}^* U_{12}^* U_{22} e^{i(\omega_2 - \omega_1)t}$$

$$\Xi = \begin{pmatrix} \frac{1}{\sigma_1} & 0 & 0 & 0 \\ 0 & \frac{1}{\sigma_2} & 0 & 0 \\ 0 & 0 & \frac{1}{\sigma_3} & 0 \\ 0 & 0 & 0 & \frac{1}{\sigma_4} \end{pmatrix}$$

- covariance matrix for random variables

average over the stochastic space-time fluctuations

$$\langle e^{i(\omega_1-\omega_2)t} \rangle \equiv \int d^4a \, \exp(-\vec{a} \cdot \Xi \cdot \vec{a}) e^{i(\omega_1-\omega_2)t} \frac{\det \Xi}{\pi^2}$$

$$\langle e^{i(\omega_1-\omega_2)t} \rangle \propto e^{-(\ldots)t} e^{-(\ldots)t^2}$$

- combined time evolution

Lindblad-type

No energy dependence

$$P_{\nu_\mu\to\nu_\tau} = \frac{1}{2}\sin^2(2\theta_{23})\left[1 - \exp(-5\cdot 10^9\gamma_0 L)\cos\left(\frac{2.54\Delta m^2}{E}L\right)\right]$$

Inversely proportional to energy

$$P_{\nu_\mu\to\nu_\tau} = \frac{1}{2}\sin^2(2\theta_{23})\left[1 - \exp(\frac{-2.54\gamma^2_{-1}L}{E})\cos\left(\frac{2.54\Delta m^2}{E}L\right)\right]$$

Proportional to energy squired

$$P_{\nu_\mu\to\nu_\tau} = \frac{1}{2}\sin^2(2\theta_{23})\left[1 - \exp(-5\cdot 10^{27}\gamma_2 E^2 L)\cos\left(\frac{2.54\Delta m^2}{E}L\right)\right]$$

γ_0 [eV] $\quad$ γ^2_{-1} [eV^2] $\quad$ γ_2 [eV^{-1}]

Gravitational MSW

$$P_{\nu_\mu \to \nu_\tau} = \frac{1}{2} - \exp(-\kappa_1)\frac{\cos^2(2\theta_{23})}{2} - \frac{1}{2}\exp(-\kappa_2)\cos\left(\frac{2.54\Delta m^2}{E}L\right)\sin^2(2\theta_{23})$$

No energy dependence

$$\kappa_1 = 5 \cdot 10^9 \alpha^2 L \sin^2(2\theta); \;\; \kappa_2 = 5 \cdot 10^9 \alpha^2 L(1 + 0.25(\cos(4\theta) - 1))$$

$$\kappa_1 = 2.5 \cdot 10^{19} \alpha_1^2 L^2 \sin^2(2\theta); \;\; \kappa_2 = 2.5 \cdot 10^{19} \alpha_1^2 L^2(1 + 0.25(\cos(4\theta) - 1))$$

$$\kappa_1 = (5 \cdot 10^9 \gamma_1^2 L + 2.5 \cdot 10^{19} \gamma_2^2 L^2) \sin^2(2\theta);$$
$$\kappa_2 = (5 \cdot 10^9 \gamma_1^2 L + 2.5 \cdot 10^{19} \gamma_2^2 L^2)(1 + 0.25(\cos(4\theta) - 1))$$

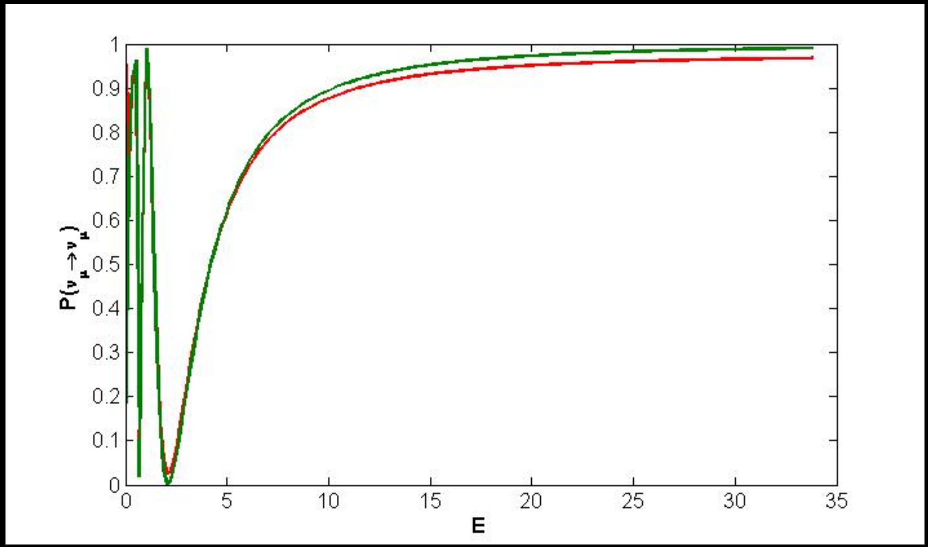
Proportional to energy

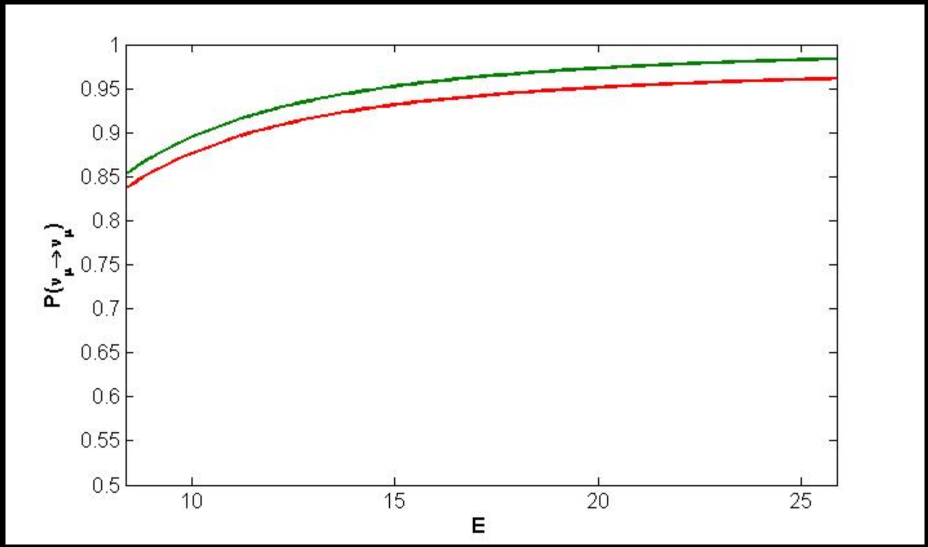
$$\kappa_1 = 5 \cdot 10^{18} \beta^2 EL \sin^2(2\theta); \;\; \kappa_2 = 5 \cdot 10^{18} \beta^2 EL(1 + 0.25(\cos(4\theta) - 1))$$

$$\kappa_1 = 2.5 \cdot 10^{28} \beta_2^2 EL^2 \sin^2(2\theta); \;\; \kappa_2 = 2.5 \cdot 10^{28} \beta_2^2 EL^2(1 + 0.25(\cos(4\theta) - 1))$$

$$\kappa_1 = (5 \cdot 10^{18} \gamma_1'^2 EL + 2.5 \cdot 10^{28} \gamma_2'^2 EL^2) \sin^2(2\theta);$$
$$\kappa_2 = (5 \cdot 10^{18} \gamma_1'^2 EL + 2.5 \cdot 10^{28} \gamma_2'^2 EL^2)(1 + 0.25(\cos(4\theta) - 1))$$

- Modify the standard oscillation formula including damping factors
- Generate fake data with standard neutrino oscillation formula
- Calculate c^2
- Qualitatively, we observe both the spectral distortion and suppression in the number of events, if there is decoherence, in addition to the conventional oscillations





CNGS

CERN-SPS $\xrightarrow{\nu_\mu \qquad\qquad \nu_\mu \;\; \nu_\tau}$ **OPERA**

L=732 km **<E$_n$>=17 GeV; 4.5$\times 10^{21}$ pot/year** **5 years** **2kT photo-emulsion**

Measure ν_μ spectrum by reconstructing μ from CC events

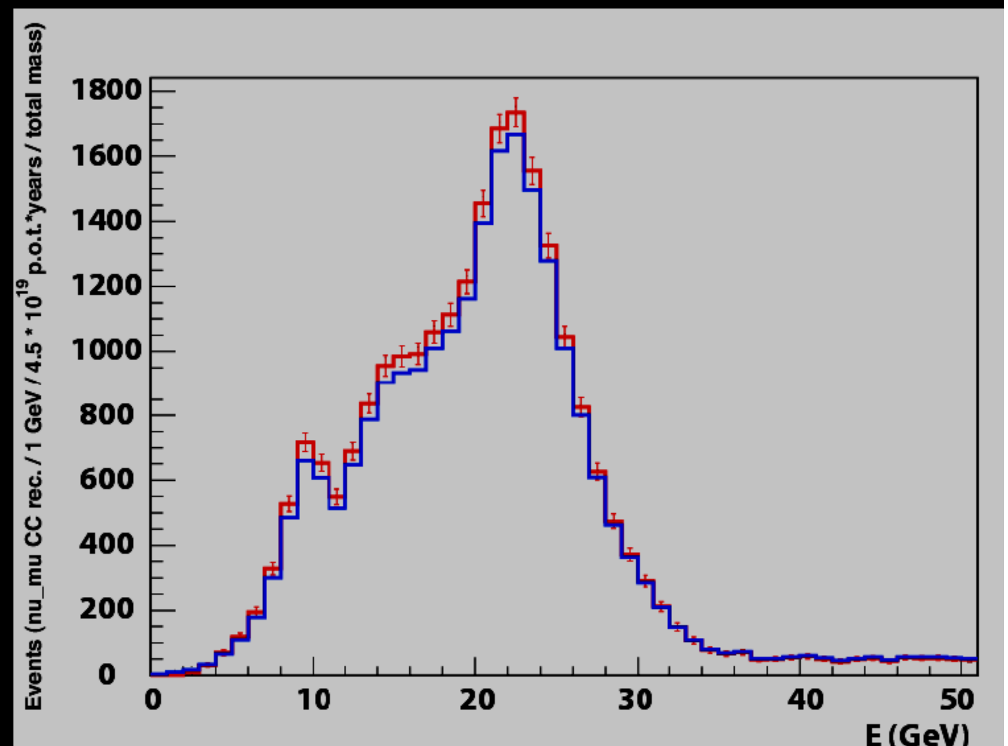
$$\frac{dN_{\mu\mu}}{dE} = A_{\mu\mu}\frac{d\phi_{\nu_\mu}}{dE}P_{\nu_\mu\to\nu_\mu}\sigma^{CC}_{\nu_\mu}(E)\epsilon_{\mu\mu}$$

$$\epsilon_{\mu\mu} = 93.5\% \quad \tilde{\sigma} = 0.2 \quad \Delta E = 20\%$$

$$\chi^2 = \sum_i [x_i - aP_i]^2/\sigma_i^2 + (1-a)^2/\tilde{\sigma}^2$$

$$\Delta m^2 = 2.5\cdot 10^{-3}\text{eV}^2$$

$$\theta_{23} = 45^\circ$$



J-PARC

Tokay ν_μ → ν_μ ν_e SK

L=295 km <E_n>=600 MeV; 1.0×10^{21} pot/year 5 years

T2K- 22.5 kT water cherenkov

Measure ν_μ spectrum by reconstructing single- Cherenkov- ring μ QE and non-QE events

Near detector

$$\tilde{\sigma} = 0.05$$

Maximal oscillation point

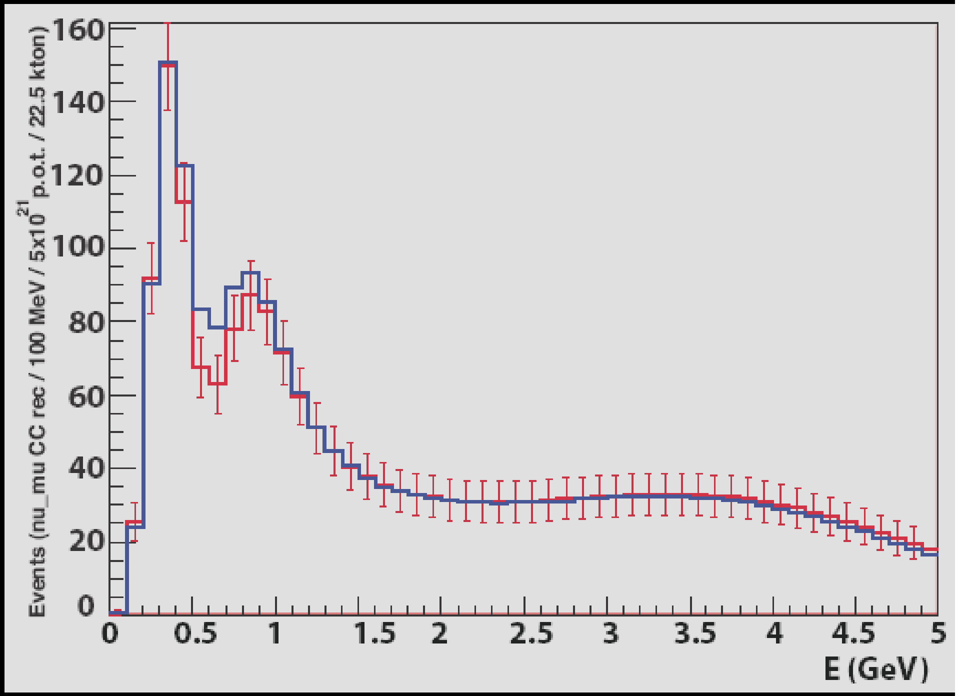
$$\tilde{\sigma}_{\text{applied}} = 0.20$$

$\epsilon_{\mu\mu} = 95\%$ $\Delta E = 20\%$

Off axis 2^0

T2KK 100 kT Argon

$\epsilon_{\mu\mu} = 95\%$ $\Delta E = 15\%$



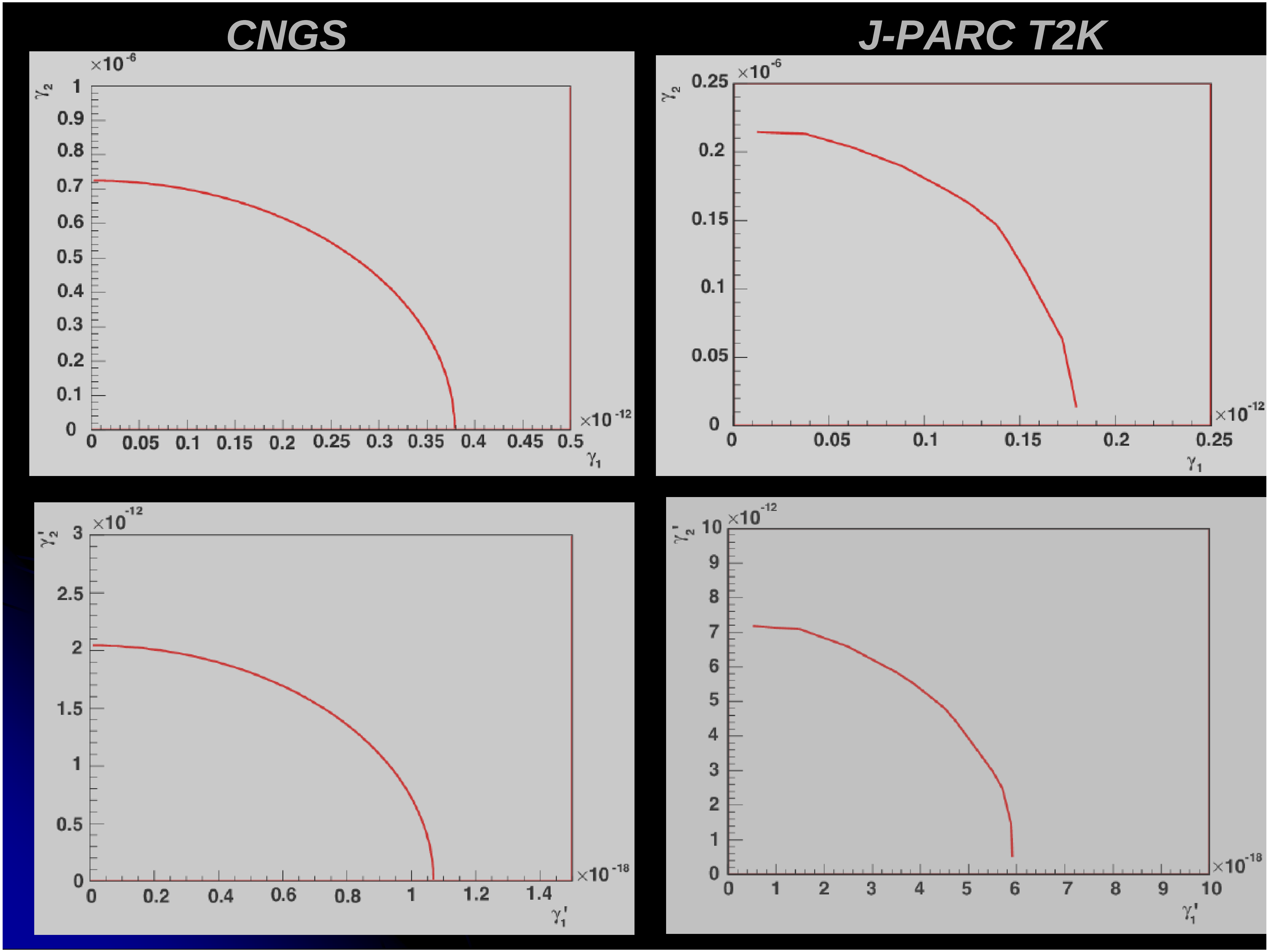
Lindblad-type	CNGS	T2K	T2KK
γ_0 [eV] ; ([GeV])	2×10^{-13} ; (2×10^{-22})	2.4×10^{-14} ; (2.4×10^{-23})	1.7×10^{-14} ; (1.7×10^{-23})
γ^2_{-1} [eV^2] ; ([GeV^2])	9.7×10^{-4} ; (9.7×10^{-22})	3.1×10^{-5} ; (3.1×10^{-23})	6.5×10^{-5} ; (6.5×10^{-23})
γ_2 [eV^{-1}] ; ([GeV^{-1}])	4.3×10^{-35} ; (4.3×10^{-26})	1.7×10^{-32} ; (1.7×10^{-23})	3.5×10^{-33} ; (3.5×10^{-24})
Gravitational MSW	**CNGS**	**T2K**	**T2KK**
α^2	4.3×10^{-13} eV	4.6×10^{-14} eV	3.5×10^{-14} eV
α_1^2	1.1×10^{-25} eV^2	3.2×10^{-26} eV^2	6.7×10^{-27} eV^2
β^2	3.6×10^{-24}	5.6×10^{-23}	1.7×10^{-23}
β_2^2	9.8×10^{-37} eV	4×10^{-35} eV	3.1×10^{-36} eV
β_1^2	8.8×10^{-35} eV^{-1}	3.5×10^{-32} eV^{-1}	7.2×10^{-33} eV^{-1}

Atmospheric

$$\gamma_0 < 0.4 \times 10^{-22}\ \text{GeV}$$
$$\gamma_2 < 0.9 \times 10^{-27}\ \text{GeV}^{-1}$$
$$\gamma^2_{-1} < 0.7 \times 10^{-21}\ \text{GeV}^2$$

Solar + KamLand

$$\gamma_0 < 0.67 \times 10^{-24}\ \text{GeV}$$
$$\gamma_2 < 0.47 \times 10^{-20}\ \text{GeV}^{-1}$$
$$\gamma^2_{-1} < 0.78 \times 10^{-26}\ \text{GeV}^2$$


CNGS
J-PARC T2K
γ2
×10^-6
γ1
×10^-12
γ'2
×10^-12
γ'1
×10^-18

Since in practice neutrino wave is neither detected nor produced with sharp energy of well-defined propagation length, we have to average over the L/E dependence etc

$$\langle P \rangle = \int_{-\infty}^{\infty} dx P(x) \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-l)^2}{2\sigma^2}}$$

$$l = \langle x \rangle$$

$$\sigma = \sqrt{\langle (x - \langle x \rangle)^2 \rangle}$$

$$x = \frac{L}{4E} \longrightarrow \langle x \rangle = \frac{\langle L \rangle}{4\langle E \rangle}$$

$$\langle 2\sin^2(\Delta m^2_{ab} x) \rangle$$

$$\langle P_{\alpha\beta} \rangle = \delta_{\alpha\beta} -$$

$$2 \sum_{a=1}^{n} \sum_{b=1,a<b}^{n} \mathrm{Re}\left(U^*_{\alpha a} U_{\beta a} U_{\alpha b} U^*_{\beta b}\right)\left(1 - \cos(2\ell \Delta m^2_{ab}) e^{-2\sigma^2 (\Delta m^2_{ab})^2}\right) -$$

$$2 \sum_{a=1}^{n} \sum_{b=1,a<b}^{n} \mathrm{Im}\left(U^*_{\alpha a} U_{\beta a} U_{\alpha b} U^*_{\beta b}\right) \sin(2\ell \Delta m^2_{ab}) e^{-2\sigma^2 (\Delta m^2_{ab})^2}$$

$$\langle \sin(2\Delta m^2_{ab} x) \rangle$$

$$\langle P_{\mu\tau}\rangle(\langle L\rangle, \langle E\rangle) = \frac{1}{2}\sin^2 2\theta \left(1 - e^{-2\sigma^2(\Delta m^2_{\mu\tau})^2} \cos\frac{\Delta m^2_{\mu\tau}\langle L\rangle}{2\langle E\rangle}\right)$$

$$\sigma = \frac{L}{4E} r$$

pessimistic $$r = \frac{\Delta L}{L} + \frac{\Delta E}{E}$$

optimistic $$r = \sqrt{\left(\frac{\Delta L}{L}\right) + \left(\frac{\Delta E}{E}\right)}$$

$$\gamma_0 L = 2\sigma^2(\Delta m^2_{\mu\tau})^2$$

$$\gamma_0 = \frac{2\sigma^2(\Delta m^2_{\mu\tau})^2}{L}$$

$$\gamma_0 = \frac{(\Delta m^2_{\mu\tau})^2 L}{8E^2} r^2$$

Atmospheric

$R \simeq 6400$ km

$d \simeq 10$ km

$$\frac{\Delta L}{L} = \frac{R}{\sqrt{R^2 \cos^2\vartheta + 2Rd + d^2}} \Delta \cos\vartheta$$

$L(\cos\vartheta = -0.95) \simeq 12000$ km $\quad \Delta L/L \simeq 0.11 \quad \Delta E/E \sim 1$

$$\sigma_{\text{atm}} \approx 3 \times 10^{-4}\ m/eV$$

$$\gamma_0^{\text{atm}} = 2 \frac{(\Delta m^2_{\mu\tau})^2}{L(\cos\vartheta = -0.95)} \sigma^2_{\text{atm}} \sim 10^{-24}\ \text{GeV}$$

GNGS

$L \simeq 1000$ km $\quad \Delta L/L \simeq 0.001 \quad \Delta E/E \simeq 0.2$

$\sigma_{\text{cngs}} \approx 2 \times 10^{-6}\ m/eV \qquad \gamma_0^{\text{CNGS}} \sim 10^{-28}$ GeV

Concluding remarks

- CNGS and J-PARC beams are very sensitive, in some particular cases, to QG induced decoherence effects
- In principle, the problem to distinguish between different dependences of damping exponents can be resolved it there are two baselines
- Damping signatures could be mimicked by uncertainties in determination of neutrino energy and propagation length
- Long baseline experiments are less affected by the risk of erroneous misinterpretations of conventional effects as a signature of decoherence

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